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Résumé.- A des densités ⩾ 4.3 × 10^{11} g/cc, la matière contient probablement des neutrons à la fois dans les noyaux et à l'état libre dans un état superfluide. Selon la valeur du moment de Fermi les nucléons produisent " le trou d'énergie" (nécessaire pour la superfluidité) par une interaction $^1S_0$ (aux densités les moins élevées) ou $^3P_2$ (aux plus hautes densités). Au-dessus de (2-2.4) 10^{14} g/cc les protons deviennent supraconducteurs, au moment où les noyaux se dissolvent. Les électrons ne deviennent pas superfluides aux températures auxquelles on s'intéresse, mais un condensat de pions peut apparaître aux densités plus élevées.

Protons et neutrons forment des réseaux de vortex quantifiés. Les premiers, en réponse à la rotation de l'étoile, les seconds en réponse au champ magnétique. La structure détaillée de ces réseaux dans l'étoile dépend en promier lieu de la température et des profils de densité ainsi que du passé cinématique de l'étoile. La dynamique des vortex fait intervenir les modes cohérents des vortex libres ("ondes sonores"), l'accrochage des axes des vortex sur les noyaux ou leur insertion interstitielle dans le réseau de noyaux et la diffusion des axes des vortex par les sites d'accrochage et par les composants chargés de la matière stellaire.

Cette dynamique peut être décrite en termes de particules ponctuelles tant que les vitesse et les forces varient sur des échelles plus grandes que les dimensions d'un vortex et tant que les effets de la frontière sphérique sont petits. Dans la phase $^3P_2$ la dégénérescence du niveau à deux neutrons J = 2 peut conduire à une plus grande complexité dynamique. Les conséquences observationnelles et la confirmation de la superfluidité dans les étoiles à neutrons (pulsars comme sources X) peuvent être, entre autres : i) les échelles de temps macroscopiques après les "glitch", résultant du couplage entre composantes " normale" et superfluide ; ii) les "glitches" causés par les décrochages des vortex ou par des brisures de croûte dues aux vortex accrochés ; iii) la modulation possible à long terme de la période de rotation par les modes cohérents des vortex ; et iv) les effets gyroscopiques dus à l'accrochage de la vorticité.

Abstract.- Matter at densities $\gtrsim$ 4.3 × 10^{11} g/cc is expected to contain neutrons both inside and outside of nuclei, which are in a superfluid state. At the relevant Fermi momenta, the effective gap-producing nucleon-nucleon interaction is $^1S_0$ (lower densities) or $^3P_2$ (higher densities). At densities $\gtrsim$ (2-2.4)× 10^{14} g/cc the protons become superconducting too, as nuclei dissolve. Electrons are not expected to be superfluid at the relevant temperatures but a pionic Bose condensate may appear at higher densities.

Both neutrons and protons form quantized vortex arrays : The first - in response to the stellar rotation, the second - in response to its magnetic field. The detailed structure of these arrays throughout the star depends primarily on the temperature and density profiles and on the kinematic history of the star. Vortex dynamics include coherent ("sound waves") modes of free vortices, pinning of vortex cores onto nuclei or as interstitials in the nuclear lattice and scattering of vortex cores by pinning sites and by the charges stellar components. This dynamics can be described as the dynamics of point particles, as long as velocities and forces vary only on scales larger than the size of vortex cores and as long as effects of the spherical boundary are small. In the $^3P_2$ phase, the degeneracy of the J = 2 two-neutron state may lead to further dynamical complexity.

Observational consequences - and confirmation - or the interior superfluid state of neutron stars, both as pulsars and as X-ray sources, can be, among others : (i) The macroscopic "post-glitch" time scales, resulting from coupling between "normal" and superfluid components ; (ii) "glitches" due to unpinning events or to crust breaking by pinning vortices ; (iii) Possible long term modulation in rotation period, resulting from vortex coherent modes ; and (iv) Gyroscopic effects of pinned vorticity.

1. Introduction.- Neutron stellar interiors form an ideal low temperatures physics laboratory : Even temperatures of $\gtrsim 10^8$ K, to which they cool very rapidly, are much lower than the many-body temperatures involved, such as the electron Fermi temperature or average Coulomb interactions of lattice nuclei. The latter are in the $\gtrsim$ MeV (10^{10} K) regime already at small depths inside the star. Low temperature phenomenae like quantum degeneration, solidification and superfluidity are thus expected to be the rule inside these stars. To some extent, they should exhibit a behaviour similar to that of terrestial materials in the 10^{-3} K regime.

Theoretical investigations of superfluidity in dense matter can currently find their best available applications in the observed macroscopic behaviour of neutron stars. In this talk we shall concentrate on the possible role played by the superfluid neutrons in the dynamics of neutron stars. Superfluid neutrons, which exist both in the core and the crustal regions, are likely to have the most important effects. Other superfluids may exist as well : Protons in the core could exhibit superconducting properties and one may also expect an interior pion condensate. The electrons will, however, probably be normal throughout the star, due to their very low transition temperatures.

In previous talks /69/ we heard about the internal structure of neutron stars as we currently understand it. We have seen, that following a rather thin surface, whose structure may depend on temperature and magnetic fields, the neutron crust commences at a density of $\sim 10^6$ g/cc. the outer crust region, where nuclei, ranging from ^{56}Fe to ^{118}Kr, are held by Coulomb forces, turns into the inner crust at 4.3×10^{11} g/cc. The inner crust essentially consists of a lattice of proton clusters, containing a magic number of protons, and of neutrons, mostly superfluid, which divide their wave functions between the space inside the proton clusters and, with lower density, the space between them. The neutron number densities exceed those of the protons, on the average, by factors of $\lesssim$ 30-40.

The core region starts at (2-2.4) $\times 10^{14}$ g/cc and contains mainly superfluid neutrons and a smaller number of protons. It is in the interior core that other phases, such as pion condensate or a neutron-hyperon solid, may develop. For a TI 1.33 M_0 neutron star, whose radius is 16.1 km, the core crust boundary is at $\sim$ 11 km radius [assuming that crust melting occurs at 2×10^{14} g/cc]. The outer-inner crust boundary lies at $\sim$ 15 km.

In section 2, I summarize the onset of superfluidity in neutron stars and the forms it takes in various regions, based on the above descrisption.

The coupling of superfluids to normal matter occurs sometimes in a striking way, when the superfluid splits-up into quantized vortex lines. Whenever superfluidity is associated with formation of bound Fermion pairs, the cores of their vortex lines becomes normal too. Vortices form whenever a non-zero circulation is imposed externally on one of the dynamic variables of the superfluid, be it rotation of the normal fluid, which imposes a velocity circulation, or, in case of a charged superfluid, an external magnetic field. Neutron stars both rotate and have large magnetic fields, so that their superfluid dynamics is likely to be controlled by such quantized vortices. I discuss this dynamics in section 3-5. First, a simple form for the general dynamical equations governing vortex motions is discussed ; then, I discuss the dynamics of the (free) core vortices, their collective motions and their response to slowing-down or spinning-up of the star ; and, finally I discuss the way by which proton clusters pin neutron vortex cores onto themselves or to the region between them and thus influence the dynamics of the superfluid neutrons.

The macroscopic behaviour of the neutron star, as a rotating container holding a superfluid, can provide a handle on verifying some of the theoretical predictions regarding the very existence of superfluids and their dynamics. Section 6 describes four of these observations : "Post-glitch" behaviour, the glitches themselves, modulations in pulsar frequencies and the gyroscopic effect on free precession.

2. The Neutron Superfluid Gap.- First suggestions on the superfluidity of neutron stellar matter were made in 1959 /1,2/ ; these were later taken up by many investigators /3-9/, who used better data and better numerical techniques to compute the superfluid gap energy as a function of density both for neutrons and protons.

Behind these suggestions was the fact that nucleon-nucleon scattering data reveal positive phase shifts -i.e. attractive interaction of two nucléons- in certain energy regimes. The largest positive phase shift at low energies occurs in the $^1S_0$ state (anti-parallel spins), while for center-of-mass energies $\gtrsim$ 150 MeV, the $^3P_2$-$^3F_2$ coupling (parallel spins) takes over as the most attractive interaction. These effective attractions do not cause the formation of a stable free neutron molecule ; however, they do cause a modification in the ground state energy of a neutron gas, as follows from the BCS model /10/ for superconductivity.

Non-interacting neutrons (or protons) fill all energy states up to the Fermi energy E_F (and Fermi momentum $\hbar k_F$, $k_F = (3\pi^2 n)^{1/3}$ where n is the neutron (or proton) number density). When interactions are introduced, only neutrons around E_F are affected, because the others do not have nearby empty states to scatter into. If, therefore, the effective interaction around E_F is attractive (note that $E_F = {}^1/_4 E_{\text{center-of-mass}}$), neutrons will form pairs, so called Cooper pairs, and an energy gap will develop around E_F, of magnitude

$$\Delta \sim E_0 \exp(-1/N(0)V) \qquad (1)$$

[E_0 is the energy range within which the neutron interaction is attractive (of the order of 10 MeV), $N(0)$ $\left(\equiv \frac{m_n k_F}{2\pi^2\hbar^2}\right)$ is the density of states at the Fermi energy for a neutron with given spin direction and V is the matrix element for the BCS effective interaction]. N(0)V turns out to be of order $\lesssim$ unity for neutron stellar densities /2/, so that Δ is expected to be of order $\gtrsim$ 1 MeV.

Similar superfluid gaps are expected to exist in pure neutron-or pure proton-matter. However, in neutron stellar interiors, neutrons differ from protons /9/: The proton effective mass is lower than that of neutrons having the same number density because of the neutron excess and because, on the average, neutron-proton interactions are more attractive than both the average neutron-neutron or proton-proton interactions. This leads to a lower gap for the protons, even when normalized to the same number densities. Naturally, number densities of protons are lower in the neutron stars to start with. Polarization of the neutron medium is the most important effect that could raise somewhat the gap for protons. Pions, if appearing in neutron stellar cores, will likely appear as a Bose-Einstein condensate of negatively charged bosons in their lowest state -- a Bose superconductor /11/. The difference between the neutron and proton chemical potentials never becomes large enough to permit spontaneous formation of pions if these only interact with nucleons in the S channel. However, other interaction channels, like the P channel, could sufficiently lower the mass of quasi-particles with the pion quantum numbers to allow for their appearance. We shall not discuss pion condensates here any further, and the reader is referred to reference /11/ for further details.

The state of a paired-Fermion superfluid is described by an order parameter $\underline{\underline{\Delta}}(\underline{k})$ in wave-vector $\underline{k}$ space. $\underline{\underline{\Delta}}$ is, in general, a 2 × 2 matrix in the spin indices of either paired Fermion /12-14/. The gap energy $\Delta(\underline{k})$ is given by

$$\Delta^2(\underline{k}) = {}^1/_2 \; \mathrm{Tr}(\underline{\underline{\Delta}}^+(\underline{k})\underline{\underline{\Delta}}(\underline{k})) \qquad (2)$$

$\underline{\underline{\Delta}}(\underline{k})$ is isotropic in $\underline{k}$ for j = 0 states, such as ${}^1S_0$ or ${}^3P_0$. Pairing with nonzero values of j results, however, in an anisotropic gap. For ${}^3P_2$ pairing one finds, in general /15,16/,

$$\Delta^2(\underline{k}) \propto \underline{k}\cdot\underset{\approx}{A}\cdot\underline{k} \qquad (3)$$

where $\underset{\approx}{A} = \underset{\approx}{B}^t\underset{\approx}{B}$ and $\underset{\approx}{B}$ is a traceless, symmetric, real 3 × 3 matrix. If only the $m_j = \pm 2$ states are paired (this fixes the direction $\underline{Z}$ of the quantization axis) then $\underset{\approx}{B}_{-1} = |{}^1{-1}_0|$, while if one only pairs $m_j = 0$ states, then $\underset{\approx}{B}_1 = |{}^1 1_{-2}|$. [Quite generally, any $\underset{\approx}{B} = |{}^1\alpha_{-(1+\alpha)}|$ with real α will represent a consistent coupling scheme]. The resulting angular dependeces in $\underline{k}$ are, correspondingly, $\sin^2(\underline{k},\underline{Z})$ and $1{+}3\cos^2(\underline{k},\underline{Z})$. A general pairing mode could give more complex angular dependence, but it can be shown that nodeless solutions to the gap equation always have lower energies than solutions with nodes. $\underset{\approx}{B}_1$ is indeed believed to be the best current estimate for the lowest energy pairing mode for nuclear matter. Stability considerations require the local $\underline{Z}$ direction to coincide with the radial direction in the star.

Generally speaking, gap energy calculations, dependent as they are exponentially on the effective interaction [(1)], are very sensitive to small details of the latter and should be regarded with caution. Nevertheless, it is instructive to examine results of some careful and systematic calculations. Figure 1 shows superfluid transition temperatures k_BT_c [$\equiv \frac{\Delta}{1.76}$ for ${}^1S_0$ pairing, $\equiv \frac{\Delta(\underline{k}||\underline{z})}{2.4}$ for ${}^3P_2$ pairing] for neutrons and protons in a neutron star as a function of total baryon density.

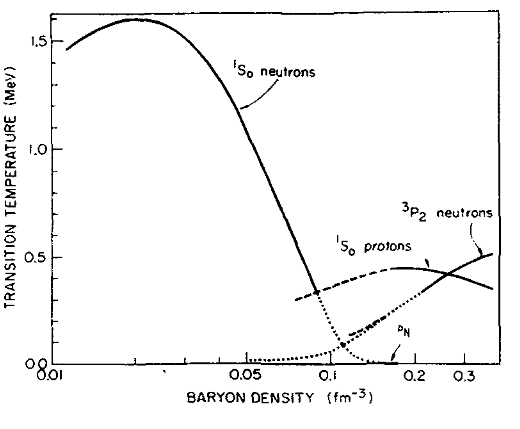

Fig. 1 : The superfluid transition temperature in neutron stellar matter. From {70}. Dotted lines roughly indicate regions of higher relative uncertainty in T_c.

For neutrons, the ${}^1S_0$ gap is seen to maximize [at value ∿ 2.8 MeV] around a density of ∿ 2.8 × 10^{13}gcm^{-3}. The transition temperature for the ${}^3P_2$ gap becomes more important around ∿ 2 × 10^{14} gcm^{-3} [at value ∿ .18 MeV for the ${}^1S_0$ gap, .24 MeV for the $\underline{k}||\underline{Z}$ ${}^3P_2$ gap].

Translation of figure 1 to radial distance inside a given neutron star requires knowledge of its density profile. For a TI 1.33 $M_\odot$ neutron star /17/ (R ∿ 16.1 km) one finds ${}^1S_0$ neutron superfluidity starting at ∿ 14.5 km and going down to ∿ 11 km, where ${}^3P_2$ neutron superfluidity commences. The transition region between the two superfluids is (quite accidentally) close to the crust-core boundary. There is also where the proton ${}^1S_0$ superfluidity commences.

While in the crust, neutrons spend some of their time inside the proton cluster /18/. Any realistic neutron gap calculation should take into account the inhomogeneous proton distribution ; as yet, no such detailed calculation has been carried out of this "proximity-effect" /19/. As a first approximation, one lets the Fermi energy and Δ vary with the changing neutron density. The accuracy of such approximation can be appreciated by comparing /20/ $\Delta(\rho)$ from figure 1 with the pairing term /21/

$$\Delta \sim 12A^{-1/2} \text{ MeV} \qquad (4)$$

in the mass formula for a finite nucleus of baryon number A. Such comparison is valid at low densities, where $\Delta(\rho) \propto \rho$. Only openshell neutrons contribute to superfluidity in nuclei ; assuming the open shell degeneracy to be $\sim(^1/_2 A)^{1/2}$, one finds the density of "free" neutrons to be $\sim \rho_N (2A)^{-1/2}$, where ρ_N is the nuclear matter density, 2.8×10^{13} g/cc. This indeed recovers (4) from figure 1.

Another relevant piece of physics related to this first approximation has to do with the dimensions of Cooper pairs. The pair wave-function consists of components at wave vectors corresponding to an energy interval of order Δ around E_F: The width of the wave packet turns out to be $\delta k \sim \frac{\pi\Delta}{(\partial E/\partial k)_{E_F}} \sim \frac{\pi \Delta k_F}{2E_F}$.

The spread in coordinate space is, therefore, $\xi \sim \frac{1}{\delta k} \sim \frac{2\, E_F}{\pi k_F \Delta} \sim \frac{\hbar^2 k_F}{\pi \Delta m}$, and this is also the superfluid coherence length. Substituting relevant numbers for neutrons, we find

$$\xi_n \sim 10^{-12} \left(\frac{\rho}{\rho_N}\right)^{1/3} \quad \left(\frac{\Delta}{1 \text{ MeV}}\right)^{-1} \text{ cm.} \qquad (5)$$

Figure 2 gives an idea of the density distributions on the scale of the proton clusters. We note that the cluster radii, R_N, are comparable to ξ so that one already expects important proximity corrections to the variability of Δ with the distance.

The ground state energy of the $^1S_0$ superfluid, the main crustal component, is

$$\varepsilon = (-)\ \frac{3}{8}\frac{\Delta^2}{E_F} \qquad (6)$$

per neutron relative to the unpaired, normal, neutron fluid. Figure 3 shows essentially differences in that energy as a function of density, and one may get an idea on their variation in space by combining figures 2 and 3.

The transition between the $^1S_0$ and $^3P_2$ neutron superfluid may, or may not be, a smooth one /22/. Apart from the uncertainties in figure 1, especially at the small T_c values near the transition point, this will depend mostly on the temperature inside the star (which, at these depths, is fairly constant with radius due to the high thermal conductivity).

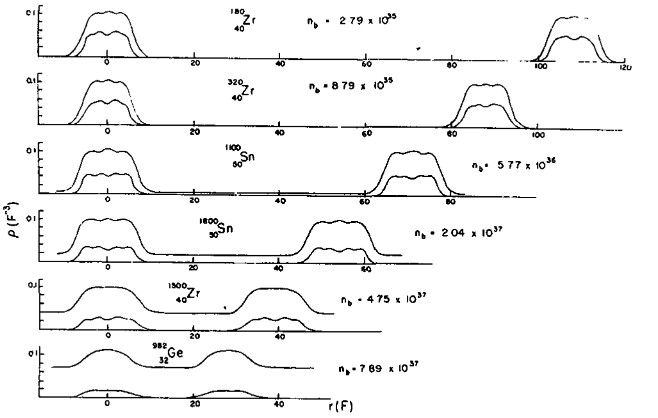

Fig. 2 : Density profiles of neutrons (upper curve) and protons inside the neutron stellar crust. n_b is the average nucleon density in nucleon /cc. The distance r is measured along the line of centers of adjacent (bcc) unit cells and is given in Fermis. ρ is given in nucleons per Fermi cubed. From {18}.

Superfluidity will be destroyed altogether if $T \gtrsim \frac{1.8 \text{ MeV}}{k_B} \sim 1.7 \times 10^{10}$ K, which is unlikely in realistic neutron stars. The transition region will become normal when $T > \frac{0.1 \text{ MeV}}{k_B} \sim 10^9$ K.

3. <u>Vortex Structure and Dynamics</u>.- Superfluid neutrons are likely to respond to the rotation of their container in the same way He II does-by forming vortex lines. In their cores, velocities reach the critical velocity which provides sufficient kinetic energy to locally overcome the gas energy Δ. Superfluid protons respond similarly to the magnetic field in which they are embedded, with one difference : when charge is involved, there is an additional length scale in the problem, the field penetration depth $\lambda = \left(\frac{mc}{4\pi n_p e^2}\right)^{1/2}$. Laboratory superconductors either repel the magnetic field altogether (when $\xi<\lambda\sqrt{2}$, type I superconductors) or form vortices which carry the field lines in their cores at strength of the critical field, with sufficient magnetic energy to locally overcome the gap (when $\xi>\lambda\sqrt{2}$, type II superconductors). However, neutron stars freeze with a strong internal field, whose diffusion time scale across the star is extremly long /23,24/ ; hence, even if the protons were type I, they would still

"make room" for the field by forming alternate normal and superfluid regions. Actually, protons seem to be type II in neutron stars /25/, and one expects proton vortices to appear, forming a vortex lattice. The lattice will likely be triangular, with nearest neighbor distance of $d_p \sim 2.5 \times 10^{-10}\ B_{12}^{-1/2}$ cm (B_{12} is the magnetic field in units of 10^{12} gauss). Since each vortex carries a flux quantum of $\phi_0 = \frac{hc}{2e}$, the relevant critical field, whenever superconductivity disappears, is $H_{c2} \sim \frac{\phi_0}{2\pi\xi_p^2} \sim 3 \times 10^{16}$ gauss.

Henceforth we shall only discuss the neutrons. We shall frequentely assume that the superfluid neutrons can be described as being in a rotating, cylindrical container of radius R.

A straight line $^1S_0$ superfluid neutron vortex has a core of dimensions $\sim\xi_n$ and superfluid velocity field $v_s = \frac{\zeta}{2\pi r}$ outside it, where r is the cylindrical distance from the vortex center and v_s is tangential. The vorticity ζ is quantized to be $\zeta = \oint \underline{v}_s \cdot d\underline{r} = \frac{h}{2m_n}$. The energy cost E_V per unit length for forming a vortex core is $\frac{3}{8}\frac{\Delta^2}{E_F} n$, where n is the number of neutrons in the core, $\sim\frac{1}{m_n}\pi\xi^2\rho_n \sim \frac{1}{3\pi}\xi^2 k_{Fn}^3$ per unit length. There is additional energy coming from the velocity field, which is $\frac{\pi\rho\zeta^2}{4\pi 2}\int_\xi^R \frac{dr}{r} = (\frac{1}{4\pi})\rho\zeta^2 \log(\frac{R}{\xi})$ and is of the same order of the core energy. The angular momentum associated with a vortex p centered at distance r_p from the center of the container is $L_{vp} = \frac{1}{2}\zeta\rho\ (R^2-r_p^2)$ per unit length.

Neutron vortices appear because the superfluid tries to keep as small a velocity difference as possible with the normal components. On minimizing $E_v - \Omega L_v$, which seems to be the relevant superfluid free energy, one finds the value of Ω, Ω_{c1}, in which the first vortex appears. This turns out to be $\Omega_{c1} \sim \frac{h}{2m_n R^2}\log(R/\xi_n) \sim 10^{-14}\ \mathrm{sec}^{-1}$. For $\Omega \gg \Omega_{c1}$, the number, N_v, of quantum vortices is given by the total vorticity, $2\pi\Omega R^2 = N_v \frac{h}{2m_n}$. Hence, the number density of vortices per unit are, n_v, is

$$n_v = \frac{2\Omega}{\zeta}\ . \qquad (7)$$

All vortices have a single unit of quantum vorticity ; multiply quantized vortices cost much higher energies. The minimum energy configuration is a triangular two dimensional lattice, of neighbor distance

$$d_n \sim 3\times10^{-2}\ (\frac{\Omega}{\mathrm{sec}^{-1}})^{-1/2}\ \mathrm{cm}. \qquad (8)$$

When $d_n \sim \xi_n$, superfluidity disappears ; this should happen when $\Omega \sim \Omega_{c2} \sim 10^{20}\ \mathrm{sec}^{-1}$ and there is therefore no danger of that ever happening in neutron stars. Observed neutron stars have, thus, a very rarefied vortex lattice.

Any force exerted on the superfluid will have to use the normal vortex core as a handle, because of the energy gap in the rest of the superfluid. Whenever physical quantities do not change appreciably over one coherence length, this property of the superfluid renders its equation of motion to be particularly simple :

Take the curl of the hydrodynamic equation of motion

$$\frac{\partial \underline{v}_s}{\partial t} + (\underline{v}_s \cdot \underline{\nabla})\underline{v}_s = -\frac{1}{\rho}\nabla P + \frac{1}{\rho}\underline{F}$$

(where P is the global pressure and $\underline{F}$ the force) to obtain

$$\frac{\partial \underline{\zeta}}{\partial t} = \mathrm{curl}\ \left|\frac{1}{\rho}\underline{F} - \underline{\zeta}\times\underline{v}_s\right| \qquad (9)$$

where

$$\underline{\zeta} = \mathrm{curl}\ \underline{v}$$

[in (9), $\rho\underline{\zeta}\times\underline{v}_s$ is the Magnus force, the analogue of the Coriolis force ; any $\underline{F}$ must balance both it and the $\frac{\partial \underline{\zeta}}{\partial t}$ term]. Now write

$$\underline{\zeta} = \underline{k}\zeta \sum_p \delta^{(2)}(\hat{\underline{r}} - \hat{\underline{r}}_p)$$

where $\underline{k}$ is a unit vector parallel to the cylinder axis, $\delta^{(2)}$ is the core vorticity distribution, approximated to be a 2-D Dirac delta function, and $\hat{\underline{a}}$ denotes a 2-D vector in the plane perpendicular to $\underline{k}$, of components a_x and a_y. We shall often use complex notation ($i=\sqrt{-1}$),

$$\hat{a} = a_x + i a_y \equiv a\ e^{i\phi}$$

$$\hat{a}^+ = a_x - i a_y \equiv a\ e^{-i\phi}$$

Now write for the (xy) component of the force the expression

$$\hat{\underline{F}} = \rho\zeta \sum_p \hat{\underline{f}}_p \, \delta^{(2)}(\underline{r}-\underline{r}_p) \qquad (10)$$

$\rho\zeta\hat{\underline{f}}_p$ is the force per unit length acting on the vortex. Substitute in (9) to find the equation of motion

$$\frac{d}{dt}\hat{\underline{r}}_p = \hat{\underline{v}}_{sp} + \underline{k} \times \hat{\underline{f}}_p \qquad (11a)$$

where $\hat{\underline{v}}_{sp}$ is the superfluid velocity at the core of the p vortex (not due to the velocity field of p). Alternatively,

$$\frac{d}{dt}\hat{r}_p = \hat{v}_{sp} + i\hat{f}_p \qquad (11b)$$

(11a) and (11b) show the general pattern of vortex motion : The vortex moves with the superfluid velocity unless there is a force acting, in which case it moves perpendicular to the force, with an additional velocity which is proportional to the force strength. They must be supplemented by a relation between $\hat{\underline{v}}_{sp}$ and the $\hat{\underline{r}}_p$. [In addition, a component of $\underline{v}_s$ may exist which does not originate in any vorticity, but we shall always ignore it].[†] The velocity field at $\hat{r}$ due to a quantum vortex at $\hat{r}_p$, in an infinite medium is given by

$$\hat{v}_s = i\xi(\hat{r}^+ - \hat{r}_p^+)^{-1} \qquad (12)$$

where $\xi = \zeta/2\pi = \frac{\hbar}{2m_n}$.

Let me also write down here, for future purposes, two other velocity fields, those due to a vortex p outside or inside of a cylinder of radius R and density ρ_i : Let the medium density be ρ. On fixing the origin at the center of the cylinder, the velocity field for the external vortex is given outside by that of the given vortex plus two image vortices inside the cylinder :One, of vorticity $\zeta\frac{\rho-\rho_i}{\rho+\rho_i}$, at the origin ; and another, of vorticity $\zeta\frac{\rho_i-\rho}{\rho+\rho_i}$, at $R^2/\hat{r}_p^+$. On comparing the resulting $\hat{v}_{sp}$, from (12), with (11b), we can define an equivalent central force (Bernoulli force) on the vortex p,

$$\hat{f}_{pe} = \xi\frac{\rho_i-\rho}{\rho_i+\rho}\,\frac{R^2}{r_p^2(r_p^2-R^2)}\,\hat{r}_p \quad (r_p>R) \qquad (13)$$

This force is attractive for $\rho_i<\rho$ and repulsive for $\rho_i>\rho$.

For the internal vortex, the velocity field inside is that of the given vortex plus an external image vortex at $R^2/\hat{r}_p^+$, of vorticity $\frac{\rho-\rho_i}{\rho+\rho_i}\zeta$. The velocity field outside is that of two images inside the cylinder : One, at $\hat{r}_p$, of vorticity $\frac{2\rho_i}{\rho+\rho_i}\zeta$; another, at the origin, of vorticity $\frac{\rho-\rho_i}{\rho+\rho_i}\zeta$. Again, we might write down the Bernoulli force ; it is now

$$\hat{f}_{pi} = \xi\frac{\rho_i-\rho}{\rho_i+\rho}\,\frac{\hat{r}_p}{R^2-r_p^2} \quad (r_p<R) \qquad (14)$$

In this case too, there is repulsion for $\rho_i>\rho$. attraction for $\rho_i<\rho$. Both (13) and (14) are singular at $r_p \to R$. This singularity is, however, of no physical consequence, as it disappears on intergrating over the finite core size.

To illustrate (11), consider two similar vortices, a distance D apart. As each core moves in the velocity field of the other, the vortices move in a circle around their midpoint at velocity $\frac{\xi}{D}$, hence angular velocity $\Omega_s = \frac{2\xi}{D^2}$. The superfluid motion is more complex ; we have

$$\hat{v}_s = i\xi\left[(\hat{r}^+ - \tfrac{1}{2}De^{-i\Omega_s t})^{-1} + (\hat{r}^+ + \tfrac{1}{2}De^{-i\Omega_s t})^{-1}\right]$$

On adding more and more vortices, they will move as if in rigid body rotation, as mentioned earlier, and so most of the superfluid will imitate, on the average, normal rigid rotation, while only a small part of it is really normal.

[See footnote on page (1)]

Now, attach a spring between the above two vortex cores. Their rotation will now become slower or faster depending on whether D is larger or smaller than the equilibrium length of the spring. This is how forces can alter the rate of rotation of a vortex lattice. As another example, consider how the superfluid is slowed down in a decelerating neutron star. Imagine the slower, normal, components to exert a tangential force on the neutron vortex cores. This makes the vorti-

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(1) Strictly speaking, an infinite vortex lattice cannot rotate, because all vortices are equivalent and no particular vortex can be the rotation axis. In practice, boundaries of the container provide the necessary cylindrical environment. Image vortices introduce an additional component of $\hat{v}_s = i\alpha\hat{r}$ in the regions $r \ll R$. So do any other normal components via their frictional force.

ces move outwards, which indeed reduces their density and hence, by (7), reduces Ω_s. When spinning the neutrons up by a faster normal component, the tangential force is in the opposite direction and vortices will indeed move inwards.

Consider finally the 2-D motion of a vortex in a central force field $f(r)\,\frac{\underline{r}}{r}$, when the superfluid velocity $\hat{v}_s$ is constant in the X direction. If $\hat{r}=re^{i\phi}$, we find from (11b) that during the motion

$$dr_p(f(r_p) - v_s\sin\phi_p) - d\phi_p v_s r_p \cos\phi_p = 0 \ ;$$

hence, the vortex moves on a trajectory of constant F,

$$(-)F = \int^r f(r')dr' - v_s r \sin\phi \ ^{(2)} \qquad (15)$$

F can be regarded as the (normalized) energy of the vortex. In a scattering process by a localized force, the vortex loses energy per unit length of an amount $\rho\zeta v_s\Delta y$. Summed over a whole vortex lattice, this indeed represents the change in total kinetic energy of superfluid rotation, $\Delta(\frac{1}{2}I_s\Omega_s^2)$. The energy is given to the scattering force centers. Note, that $\Delta y=0$ if the force center remains stationary, because the potential well $\int^r f(r')dr'$ is cylindrically symmetric. This is also true if the force center moves parallel to $\underline{v}_s$. In this case, one simply replaces v_s in (15) by v_s', the relative velocity between the superfluid and the force center before the scattering. Whenever $v_s'>0$ the superfluid, in our notation, moves slower than the force center and any energy given to the force center (in this moving frame) means $\Delta y<0$, namely, the superfluid speeds up. Whenever $v_s'<0$, the opposite is true. However, these changes in superfluid energy can only occur if the force center moves in the y direction during the scattering process.

Vortex structure and dynamics in neutron stars naturally differ from those of 2-D vortices in the above discussion in one important aspect--the spherical symmetry of the container. It is still an unsolved problem just what neutron vortices look like near their neutron drip end and what the proper boundary conditions there are, but several attempts have been made[26,27] to understand vortex behaviour in spherical or, generally, ellipsoidal containers. It is beyond the scope of this talk to discuss details of this dynamics, particularly since, in its overall properties, it does resemble that of the infinite cylinder. Rigid body rotation in an ellipsoidal container with impenetrable boundaries is still achieved by an equi-distant triangular array of vortices (which now diminish in length the further they lie from the rotation axis). As long as forces on the superfluid do not vary appreciably over an intervortex length scale, the motion of vortices remains quite coherent and they remain quite straight : while the effective tension in an isolated vortex of order $\sim \frac{1}{4\pi}\rho\zeta^2 Log(R/\xi_n)$ [this is the tension due to the velocity field ; a similar term will come from the core energy], the tension of a bundle of N vortices, moving in their own velocity field, is N^2 times that of an isolated vortex. Even when forces do vary along the vortex length, as they do in neutron stars, they will, *in general*, not cause any bending or twisting of vortex lines--i.e., turbulent motion--and the dynamics will be that of a vortex acted on by the average force along the vortex[27]. This, however, may not be the case when local pinning forces exist (see below).

It is worth commenting, in conclusion of this section, that $^3P_2$ superfluids may have vortex structure which is more complex than that of $^1S_0$ superfluids.[12] We have noted earlier that the general shape of the $^3P_2$ gap as a function of $\underline{k}$ is given by [(3)].

$$\Delta^2_\alpha(\underline{k}) \propto k_x^2 + \alpha^2 k_y^2 + (1+\alpha)^2 k_z^2$$

for any real α. All of the order parameters $\Delta_\alpha(\underline{k})$ are degenerate near $T=T_c$. At $T\sim 0$, which is the physical regime for neutron stars, $\underline{\underline{\Delta}}_1$ gives the lowest energy, but one might think of changing into some other $\underline{\underline{\Delta}}_\alpha$ when vortices form, depending on the total energy balance. This possibility may also come up in case of vortex pinning. Various phases in liquid He^3 are characterized by even more complex order parameters, which make it possible to introduce vorticity into the superfluid without forming normal cores.[12] $^3P_2$ pairing of neutrons is not likely to have that possibility.

4. <u>The Modes of the Free Vortex Lattice</u>.- The lowest energy stable configuration of a vortex lattice has a triangular 2-D unit cell.[28] Other configurations are intrinsically unstable and represent vortex "fluids"

---

(2) Note, that (11a) can be written as $\frac{d}{dt}\hat{\underline{r}}_p = \underline{k}\times(-\underline{\nabla}F)$.

rather than a vortex "solid". Consequently, the triangular lattice has stable modes of "phonons" or "Tkachenko" modes, in which small oscillations (in the corotating frame, rotating at frequency Ω_s) around the equilibrium lattice sites take place.

An analysis of the modes with long wavelengths yields a wave equation for these sounds waves : Denoting by $\hat{\underline{s}}(\hat{\underline{r}})$ the displacement vector of a vortex at $\hat{\underline{r}}$, one finds,[28,29]

$$(\hat{\underline{\nabla}}\cdot\hat{\underline{\nabla}})\,\hat{\underline{s}} - \frac{1}{c_T^2}\frac{\partial^2}{\partial t^2}\,\hat{\underline{s}} = 0 \qquad (16a)$$

$$\underline{\nabla} \times \underline{s} + 2\Omega_s\,\hat{\underline{\nabla}}\cdot\hat{\underline{s}} = 0 \qquad (16b)$$

where the "sound velocity" C_T is

$$C_T = 1/2\ (\Omega_s\zeta)^{1/2} \qquad (17)$$

Equation (16b) describes the special character of the vortex motion : Any change in vortex density, $\propto \hat{\underline{\nabla}}\cdot\hat{\underline{s}}$, induces a tangential motion because of the vorticity change in that neighborhood.

(16a) and (16b) have the following set of solutions for a phonon mode of wave number k and frequency $\omega = C_T k$:

$$\hat{\underline{S}}_1^{(m)} = I(t)\,e^{i\phi}\left[e^{im\phi}\,J_{m+1}(kr) + e^{-im\phi}J_{m-1}(kr)\right]$$

$$\hat{\underline{S}}_2^{(m)} = L(t)\,e^{i\phi}\left[e^{im\phi}\,J_{m+1}(kr) - e^{-im\phi}J_{m-1}(kr)\right] \qquad (18)$$

with

$$I(t) = \cos\omega t - i\,\frac{\omega}{2\Omega_s}\sin\omega t$$

$$L(t) = \cos\omega t - i\,\frac{2\Omega_s}{\omega}\sin\omega t \qquad (19)$$

and $J_v(x)$ being the Bessel function of integral order v.

Individual vortices move in ellipses with one major axis directed along $\hat{\underline{r}}$, the radius vector from the center of the cylinder to the equilibrium site of the vortex. In type 1 modes, the ellipses of the longest wavelengths modes ($\omega \ll \Omega_s$) are elongated in the $\hat{\underline{r}}$ direction ; in type 2 modes, they are elongated in the tangential direction. The overall superfluid motion is a combination of oscillatory differential rotation and tangential velocity variations.

The solutions (18) must be supplemented by boundary conditions at R. These depend on the details of the superfluid-container interaction, and are rather hard to work out microscopically. However, in investigating the way in which Tkachenko modes are reflected in the dynamics of the container, i.e. the neutron star, it is instructive to consider the change in superfluid angular momentum while oscillating at a given mode. One easily finds, from

$$L_s = \Sigma L_{vp} = \frac{1}{2}\,\zeta\rho_p \sum_p (R^2 - |z_p|^2) \qquad (20)$$

that

$$\Delta L_s = -\zeta\rho\ \mathrm{Re}\sum_p z_p \hat{S}_p^+ \propto \int r^2 dr\, d\phi\ \mathrm{Re}\,(\hat{S}\,e^{-i\phi}) \qquad (21)$$

Comparing (21) with (18) shows immediately that $\Delta L_s \equiv 0$ for $m \neq 0$; vortices on a given circle move in both directions and intrinsically conserve superfluid angular momentum.

Changes in L_s could arise only for

$$\hat{S}_2^{(0)} = aJ_1(kr)\,e^{i\phi}\left(\cos\ \omega t - i\,\frac{2\Omega_s}{\omega}\sin\ \omega t\right)$$

This is the pure differential rotation mode, and we now have

$$-\Delta L_s = 2\Omega_s R^3 \rho a\left(\frac{J_2(kR)}{kR}\right)\cos\omega t \qquad (22)$$

(per unit length)

Vortices move in ellipses which are elongated in the tangential direction.

When no coupling exists between the superfluid and the container, the relevant eigenvalues for k satisfy

$$J_2(kR) \equiv 0$$

The case of interest is when these are coupled. In neutron stars, coupling occurs throughout the star, due to the interaction of the vortex cores both with the nuclei in the stellar crust and with the interior charged fluids, which corotate with the crust since the magnetic field ties them rigidly to it. This coupling can, for example, be effectively described by adding to (11b) a force of the form

$$\hat{f}_p = -\alpha\left(\frac{d}{dt}\hat{r}_p - i\Omega_n\hat{r}_p\right) \qquad (23)$$

where Ω_n is the crustal angular frequency. In (16a), such force would add, on the r.h.s., a term (to first order in α) $2\alpha\Omega_n c_T{}^2 \frac{\partial}{\partial t}\hat{\underset{\sim}{S}}$, which represents the damping of the coherent modes ; the angular momentum exchange proceeds via (21) itself, and its magnitude can be found once (16a) and (16b) are solved for $\hat{\underset{\sim}{S}}$. Excitation of T-modes inside neutron stars can again occur via the crust-superfluid coupling, whenever the crust undergoes noisy frequency behaviour, say. The variability of α with depth in the core will determine which particular modes will be excited ; since that variability is quite small, one expects the excitation of the fundamental T-mode to be most efficient. It's period should be of the order.

$$P_T \sim 20\left(\frac{R}{10^6\ \mathrm{cm}}\right)\left(\frac{P_n}{1\ \mathrm{sec}}\right)^{1/2} \text{months} \qquad (24)$$

where P_n is the neutron stellar period. Several preliminary experiments in rotating buckets of liquid He^4 do seem to indicate similar coherent oscillation modes in the superfluid.[31]

5. Pinning of Crustal Vortices.- We have already pointed out that below the neutron drip regime in the neutron stellar crust, and down to the melting regime, neutrons are free to move outside and inside of the proton clusters, such that their density inside their clusters, ρ_i, is higher than outside them, ρ. It is the nucleon-nucleon interaction which determines the neutron density variations near the protons and the clusters'sizes, and the Coulomb interaction which controls the lattice structure of the proton clusters.

The (periodically) variable neutron density is of particular significance in our discussion of neutron vortices. Formation of e neutron vortex core requires energy to break up the Cooper pairs (see above). As this energy varies with density, the vortex core will feel an effective force,[32] derivable from that energy as a potential energy. Such force is generally called a "pinning force".

A detailed microscopic calculation of the spatial variation of vortex energy in the proton cluster lattice of a neutron stellar crust is yet to be performed. One can, however, invoke general semi-classical arguments to sketch the role which pinning forces could play in the macroscopic dynamics of neutron stars.

Two kinds of pinning forces are expected :

(i) Pairing-energy forces.- The expected energy difference between neutrons inside and outside the clusters is [from (6)]

$$\delta E = \frac{3}{8m_n}\left[v_i\rho_i\frac{\Delta^2(\rho_i)}{E_F(\rho_i)} - v\rho\frac{\Delta^2(\rho)}{E_F(\rho)}\right] \qquad (25)$$

where v_i and v are the relevant vortex core volumes. The cores radius, which is needed to calculate v_i and v, is a very important unknown : It is certainly $\xi_n(\rho)$ whenever $\xi_n(\rho) \ll \frac{\rho}{|\nabla\rho|}$, but this is not the case here, as ξ_n is sometimes of the order of the nuclear radius R_N.

From (25) we obtain an effective force $|\underset{\sim}{F}|$ on dividing δE by the smallest between ξ_n and R_N. $\underset{\sim}{F}$ can be either attractive or repulsive, and its exact spatial variation over R_N or ξ_n is again unknown.

(ii) Bernoulli forces.- These were discussed earlier [(13), (14) should be integrated over the core] and are always repulsive. [See Fig. 4]

(i) and (ii) are of the same order of magnitude, with energies of around 1 MeV per nucleus. They certainly represent only two aspects--albeit, the most important ones-- of the full quantum mechanical calculation. Furthermore, one must also consider the 3-D aspects : Vortices would have pinning sites dense along their cores if they could easily bend, namely, if gained pinning energy is higher than the vortex energy lost by stretching. Alternatively, that would occur if the Coulomb energies would permit the proton clusters to move from their equilibrium sites into the vortex cores. In any other situation, pinning will be essentially random and dilute along the vortex core, depending on just the nuclei that happen to lie along it,[32]

To make some progress, we consider a model in which the problem is still 2-dimensional, with an effective energy F (of (15)) as drawn in figure 4.[32]. Note, that F is given for a system in which the nuclei are at rest and v_s is replaced by the relative superfluid velocity, v_s'. This will well represent the strong pinning regime. Figure 3 gives the results of a calculation to determine the size of effective pairing pinning forces as a function of density inside the star.[33]

Consider first the $v'_s=0$ case. In the inter-nucleus regime there are always minima of F. These may not be sufficiently deep to introduce energy barriers--i.e., vortices may not be actually pinned to these minima. The best chance for inter-nucleus pinning-"threading"-will arise when type (i) forces are repulsive and so raise the potential barrier on the nuclei.

When type (i) forces are attractive, deep potential minima may be located on the proton clusters and, because of the Bernoulli barrier, will always produce pinning against outward motion. Vortices could still cross that barrier in both directions (but with different rates) by quantum-mechanical tunneling.

Next, consider pinning for $v'_s\neq 0$; to be specific, assume that the v_s implied by the given vortex lattice density is <u>higher</u> than the crustal rotational velocity, as will be the case in a decelerating neutron star. Then $v'_s<0$ (it is in the -x direction) and so a term $-|v'_s|y$ is added to F around a given nucleus (see Fig. 4).(3) The minimum of F in the pinning case will be shifted from y=0 to some y>0, and the height of the barrier to escape from that minimum to a still larger y will become smaller. The vortex is now pinned off-center onto the nucleus, as if it tries to move out, to reduce $|v'_s|$, but is being held by the pinning force which, acting inwards, gives the vortex an effective velocity in the +x direction to cancel v'_s. The pinning force furnishes the needed Magnus force, and vortices can escape from their pinning sites only by tunneling or thermal excitation.

As v'_s increases, $-|v'_s|y$ makes the y>0 barrier come down until it altogether disappears and there is no local minimum to F. This is the unpinning situation, and the vortex starts to move freely, at velocity $v'_s\hat{x}$ and at constant F. Typical values for $\frac{|v'_s|\max}{r}$ as a function of r inside neutron stars are shown in figure 5, both for the pinning and the non-pinning ("threading") regimes.[32]

A necessary condition for a given vortex to repin is to either move out to a region where the pinning minima are deeper and F still has a minimum even for the given $|v'_s|$; or else to move sufficiently

(3) In the global neutron star environment, y is, of course, the cylindrical r coordinate.

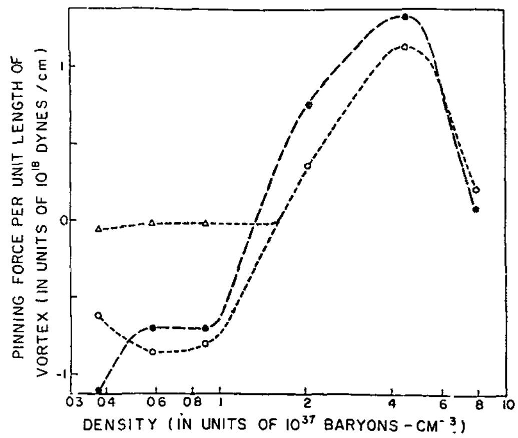

Fig. 3 : Average pinning (positive) or "threading" (negative force per unit length of a vortex line in a neutron stellar crust assuming dense pinning along the vortex. Circles use Equ.(25), with $v=v_i$, divided by Max(ξ_n,R_N), for gap values taken from {4}(black circles) or {8} (open circles). Triangles correct threading values by estimating the average interstitial force, using {8}. Points correspond to the six values of Fig.2. Adapted from {33}.

The equations of motion are still rather simple : There is a force acting on the pinning proton clusters, which causes them to move. Both the pairing energy <u>and</u> (as a simple calculation shows) the Bernoulli forces act back on the nucleus. [For example, if pairing forces vanish, a vortex and a free nuclear column will rotate around a common (external) center due to their mutual Bernoulli repulsion]. But, the complicated matter is solving these coupled equations,

$$\frac{d}{dt}\hat{r}_p = \hat{v}_{sp} + if(\hat{r}_p-\hat{r}_q) \qquad (26)$$

$$\frac{d^2}{dt^2}\hat{r}_q + \hat{G} = -\eta f(\hat{r}_p-\hat{r}_q)$$

where $\hat{r}_q$ is the complex coordinate of the q proton cluster column, closest to the vortex, $\hat{G}$ is the Coulomb force exerted on column q per unit length by all other proton clusters and η is a constant. It is clear, that once the vortex-nucleus crossing time, $\frac{R_N}{|v'_{sp}|}$, is much smaller than any proton lattice oscillation time, scattering efficiency is dominated by the recoil of single nuclei (optical phonons) ; otherwise, recoil of many proton columns

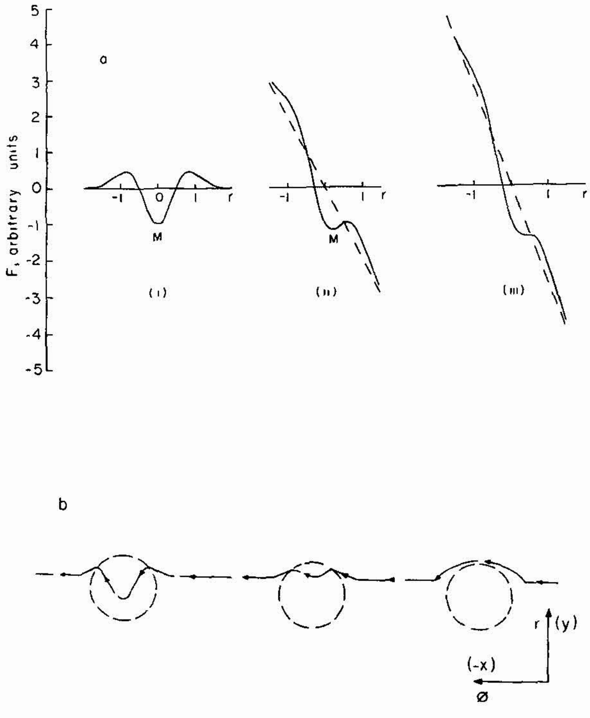

Fig. 4. A Model pinning site.

(a) Vortex potential F in the vicinity of the site for (i) $v_s'=0$; (ii) v_s' of order $(v_s')_{max}$ $\{>v_s'\}$; (iii) $v_s' = (v_s')_{max}$. The cylindrical r coordinate is on the horizontal axes and the cut is at constant ϕ. Broken lines show the hydrodynamical potential $-v_s'r$. M is the pinning position.

(b) Vortex scattering trajectories, near an immobile pinning site, viewed along the $\underline{k}$ axis. Motion at infinity is along $\underline{\Phi}$. Pinning forces vanish on the broken-line circles. For vortices coming in from the left, the figure should be viewed from above $\{t \to -t\ ,\ \underline{k} \to -\underline{k}\}$.

Note : In a "threading" regime, no minimum occurs at r=0 in (a), (i) or (ii), and trajectories never "turn around".

are involved and direct scattering is less efficient (Acoustic phonons). The electron gas, polarized by the distorted proton lattice, may also be an effective scatterer against the vortex cores. Preliminary calculations indicate that scattering of vortices towards their hydrodynamical positions is a rather fast process as long as $|v_s'|$ is large enough. When $|v_s'|$ decreases, repinning becomes more probable. Because of the spherical density profile of neutron stars it may happen, that pinning forces will only operate in one part of a vortex line, the rest of it being unpinned. It is still a somewhat open question whether those "free" vortex parts could move tangentially with the local superfluid

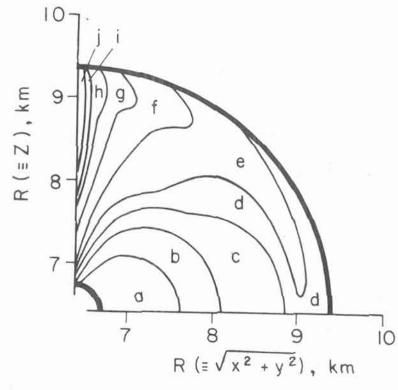

Fig. 5 : Model-calculation results for the loci of constant maximum rotational angular velocity difference, $|v_s'|_{max}/r$, above which vortex core locally unpin, inside the crust of a $\sim 0.4M_\odot$ BBP (see in ref. 10) neutron star. The cut is in a plane through the rotation, z, axis. The constants of the curves vary from $|v_s'|_{max}/r \sim 5\ sec^{-1}$ (a-b boundary) to $|v_s'|_{max}/r \sim 600\ sec^{-1}$ i-j boundary roughly by factor 2 jumps. Notice the strong off-equatorial-plane pinning. From {34}.

velocity and out by scatterings, forming a turbulent transition sheet between pinned and unpinned vorticity.[34]

It has also been suggested that volume quantum turbulence may fully develop[35] ; the vortex lattice may, however, be too rigid for that to occur : It presently seems more likely that locally unpinned vortices would move to their hydrodynamical positions by forming only thin turbulent layers. Such turbulent layers will be especially important at the core-crust boundary. There is indeed some evidence that a quantum Ekman layer will form, one intervortex distance thick and with kinematic viscosity $\zeta (\equiv \frac{h}{2m_n})$.[36] The Ekman spin-up time is $\propto (R^2/\zeta\Omega)^{1/2}$. Note, that this, in order of magnitude, is roughly the period of the lowest T-mode of the core.

As a neutron stellar crust slows down, the core vortices have no problem moving out, but on reaching the crust they pin to form a transition region from the new, lower angular frequency, Ω_s to the original Ω_s. This likely continues until Magnus forces grow above the maximum pinning. One way by which, one may speculate, the pinned superfluid could still rotate at the new angular frequency, is by forming vortices of negative vorticity, i.e., which rotate in the opposite sense. Generally, vortices can only form in a superfluid-normal fluid boundary and their formation costs, of course, additional energy. Outside of the pinned neutron regime, which is the only region where negative vortices could dynamically form in a decelerating star, there do not seem to be any normal neutrons but other normal components exist. In an accelerating star, additional positive vortices will be needed inside of the pinning region to accelerate the fluid. These may form in the $^1S_o - {}^3P_2$ transition region, if it is normal.

6. Observations and Experiments.- With a macroscopic fraction of the neutron stellar interior expected to be superfluid, one should hope that the dynamics of the outer crust, whose rotation is actually that observed in radio and X-ray pulsars, would be sufficiently influenced to reflect the interior superfluidity.

A macroscopic time scale is observed in pulsars in connection with sudden spin-ups in their rotation frequencies, the so-called "glitches".[37-44,69] Glitches are of order $\frac{\Delta\Omega}{\Omega} \sim 10^{-6}$ in Vela, $\sim 10^{-8}$ in the Crab and others ; the Vela ones are the largest known. Following the shrap rise in frequency, a slow decay of fraction Q of the jump is observed. Decay time scales τ vary from days (for the Crab pulsar) to over one year (for Vela). The fraction Q varies from $\sim .1$ (in Vela) to $\sim .95$ (Crab) and the excess deceleration during the initial stages of the relaxation is $\frac{\Delta\Omega\ \Omega}{\Delta\Omega\ \Omega} \sim 10^3$ (see reference 69). Time between glitches is of order $\sim$months to yrs.

Regardless of the cause for a given glitch, its "postglitch" behaviour can be understood in terms of a basic two-component interaction model[45] (to which more components can be added depending on the circumstances).

The model argues, that following some rapid internal event, one (or several) stellar component spins up on a short time scale, while the rest of the star follows suit after a longer (single or complex) time scale, indicative of its interaction either with the original spin-up source or with the first component(s). In dense matter, where normal interaction times are of the order of the nucleon-nucleon scattering times ($\sim 10^{-24}$ sec),[30] the large τ, finite Q and their reasonable repetitiveness in various glitches indicate that a macroscopic fraction of the star is involved, and points towards a large quenching of energy transfer, i.e., a forbidden energy gap. Since protons are charged and hence coupled electromagnetically to the crust, with very short Alfven timescales, superfluid neutrons remain the only viable candidate for the second component. The actual coupling mechanism (of the type represented by eq. (23)) could then be scattering of electrons or protons against the normal neutron cores[46], scattering of dribbling vortices against pinning sites in the crust or some other scattering mechanism ; it could also reflect an Ekman pumping time at the crust-core boundary due to the quantized vortex turbulent sheet (see p.24). The point still remains, that neutron superfluidity is strongly indicated.

Some "glitch" mechanisms themselves may have to do with superfluid neutrons. Starquakes have been suggested as one possibility,[47] they being triggered by strains developing in a crust[48,49] or solid core[50] which is slowing down. While such starquakes coulb be the cause of smaller glitches,[51] they may not be the direct cause for the Vela spin-ups : Their magnitude and frequency may require the Vela pulsar to be a stronger X-ray source than it is observed to be, if the corequake heat could not be efficiently carried away by neutrinos or non-radiatively.[52,53]

Pinned superfluidity may cause glitches because the pinning forces cause the superfluid to rotate in a metastable state when the crust slows down. Both starquakes and unpinning events result from metastable channeling of some of the pulsar slowdown energy into the nuclear-vortex lattice reservoir. An "unpinning" sudden event[54] may occur either in a region where the limiting Magnus frequency, $\frac{(V_s^{\prime})_{max}}{r}$,(4) increases outwards and so unpinning and outward motion of interior vortices will cause a chain reaction on their way out ;[32] or else it may occur in regions where the nuclear lattice elastic forces are weaker than pinning forces and local (crustal) fractures are induced by the Magnus forces, thus again causing the disruption and unpinning of vortices and their motion outwards.[22] Any motion outwards of (previously pinned) vortices transfers superfluid angular momentum to the crust at an essentially $< \frac{R}{|V_s^{\prime}|}$ timescale. The time scale will be much longer when nuclear columns diffuse outward with their pinned vortices, driven by the Magnus force, and the material flows plastically rather than break suddenly.[32] The fractional excess angular momentum reservoir in the pinned superfluid is of order 10^{-2}, (ref. 22,55) so that yearly events of size $\sim 10^{-6}$ pose no fundamental problem for Vela.

The unpinning glitch scenario is yet not fully understood, but progress has been made in the last few years to arrive at a consistent picture for the Vela glitches which will take into account both all available Vela data and their comparison with other neutron stars.[32]

T-modes, if possible in the neutron stellar core, could, as mentioned above, reflect in the crust as modulations of its rotation frequency.[29] Period variations have been observed both in Her X-1[56] and in Cen X-3[57] which have frequencies close to that predicted by (24). However, these and other X-ray sources are rather generally noisy, undergoing frequency variations of various sizes and temporal structure. Such variations can result both from variations in the external accretion torque on the stellar crust and from variations in the internal torques, associated with either periodic T-modes or random quakes and unpinning events. One must, therefore, exercise extreme care in interpreting X-ray timing data to reflect T-modes, and use a detailed statistical analysis.[58-60] Excitation of a large amplitude T-mode may cause the spin-down episodes in Her X-1, and further timing date will be able to clarify this matter.

If a large gyroscope, in the form of pinned vorticity, exists inside neutron stars, one expects it to strongly affect their free-precession properties.[55] Recall, that when free precession takes place, a corotating observer sees the rotation axis precess around a fixed axis in the star, the "elastic reference axis".[61] To an external observer, free precession is seen as periodic latitude variations of surface feactures.

Free precession is a result of misalignment between the angular velocity and angular momentum vectors in the star. It does not occur in homogenous spherical or fluid bodies, because there, these vectors are always aligned. In a partly solid, oblate, star, the frequency Ω_p of free precession is given by [61]

$$\Omega_p = \frac{3}{2}\varepsilon_w\Omega \qquad (27)$$

where Ω is the rotation frequency, and ε_w is the remnant oblateness (stellar oblateness for $\Omega \to 0$). When a gyroscope, of angular momentum ψL (L is the stellar angular momentum) is fixed in the star, misalingment is guaranteed, and (27) is modified to give [55,65]

$$\Omega_p = \psi\Omega + \frac{3}{2}\varepsilon_w\Omega \qquad (28)$$

The importance of (28) is, that it provides a lower limit, $\psi\Omega$, on Ω_p. Current estimates for crustal superfluid give $\psi > 10^{-2}$[ε_w can be as low as $\sim 10^{-7}$].

Free precession has not yet been identified in any neutron star,[62] even though several slow periods have been observed whose origins are not completely understood and may be candidates for free-precession-for example, the 35^d peridocity in Her X-1[63] or, if SS433 does turn out to be a neutron star--its 164^d periodicity.[64] In either case, that would require non-pinning, implying that either the stars are very young and hot, or that pinned superfluidity is not yet well understood, or that the crustal superfluid does manage to follow the crustal motion by creating a more complex velocity (and vorticity) field or by flowing plastically.

Note, that even if vortices were not pinned, the vortex structure would have an impact on free-precession because of the crust-superfluid coupling.[65,55]

Let me conclude by mentioning possible terrestial laboratory experiments on rotating He^4 buckets which might shed some light on the ideas described above :[31,66,67]

By studying accelerating or decelerating buckets one could follow the container-fluid relaxation, sudden unpinnings of vortex lines and T-modes. On using magnetically suspended non-spherical buckets one could study pinning via free precession, which may be pumped via a resonant electro-magnetic torque.

Of special importance would be experiments in which the pinning material is put in some intermediate location and the formation of various boundary layers while the bucket decelerates is monitored.

Several experiments in HeII have already been carried out along these lines.[31,67,68]

In conclusion.- Let me summarize and say, that the basic framework for understanding superfluid dynamics in dense matter and its implication on neutron stellar dynamics is now mostly at hand. Several interesting problems still remain open : On the theoretical side, one may want to arrive at a more detailed picture of neutron vorticity inside of a proton cluster lattice.

(4) averaged, with some weight function, along the vortex line,

On the experimental side, one should check the general picture in laboratory experiments and, with careful data analysis, look further for effects of superfluidity and vortices in confirmed neutron stars.

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