Chatter suppression in milling processes using periodic spindle speed variation

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Abstract
Spindle speed variation is a well known technique to suppress regenerative machine tool vibrations, but it is usually considered to be effective only for low spindle speeds. In this paper, the effect of spindle speed variation is analyzed in the high-speed domain, for the spindle speeds corresponding to the first flip (period doubling) and to the first Hopf lobes. The optimal amplitudes and frequencies of the speed modulations are computed using the semi-discretization method. It is shown that period doubling chatter can effectively be suppressed by spindle speed variation, while the technique is not effective for the quasi-periodic chatter above the Hopf lobe. The results are verified by cutting tests.

1 INTRODUCTION

Productivity of machining is often limited by vibrations that arise during the cutting process. These vibrations causes poor surface finish, increase the rate of tool wear and reduce the spindle lifetime. One reason for these vibrations is the surface regeneration, i.e., the tool cuts a surface that was modulated in the previous cut. The theory of regenerative machine tool chatter is based on the works of Tobias and Fishwick [1]. This knowledge initially dedicated to the turning process has been adapted to milling operations [2, 3] and led to the development of the stability lobe theory. Since then several improved models and analysis techniques have been appeared including detailed analysis of the governing delay-differential equation and time domain simulations [4-9]. These models all use the so-called stability lobe diagrams, which allows to choose the maximum axial depth of cut for a given spindle speed associated with a chatter free machining. In many practical cases, however, the choice of the optimal speed is difficult because contradictory parameters interact with productivity [10, 11].

There are different ways to reduce chatter vibrations. Classical solutions are based on the increase of the stiffness of the mechanical components and on the increase of the damping by reducing cutting speed or by adding dampers. Tools with variable pitches [12], or with variable helix angles [13, 14] can also be used to suppress chatter. The idea behind these techniques is that each flute experiences different regenerative delay, this way the regenerative effect is disturbed that may reduce the self-excited vibrations for certain spindle speeds.

A similar technique to disturb the regenerative effect and to suppress chatter vibrations is the spindle speed variation. As opposed to variable pitch or variable helix cutters, spindle speed variation can effectively be used in a wider spindle speed range, since the frequency and the amplitude of the speed variation can easily be adjusted in CNC machines even during the machining process. The idea of spindle speed variation became in the focus of interest in the 1970's. Takemura et al. [15] presented the first simple models to study the stability of variable speed machining; they predicted significant shift of the stability lobes to higher depth of cuts, but the experimental tests showed only small improvements. Sexton and Stone [16, 17] have developed a more realistic model, they found some improvements in the stability properties for low spindle speeds.

Stability analysis for variable speed machining requires special mathematical techniques, since the corresponding mathematical model is a delayed-differential equation with time varying delay. Sexton et al. [16] considered the projection of the solutions of the system to the subspace of periodic functions and used Fourier expansion to reduce the problem to an eigenvalue analysis. Tsao et al. [18] have developed a model taking the angular coordinates as variables instead of time. This approach was further improved by Jayaram et al. [19], who used a special combination of Fourier expansion and an expansion with respect to Bessel functions to analyze the system. Insperger and Stépán [20] used the semi-discretization method to construct stability diagrams for variable speed turning. They showed that the critical depths of cut can be increased for low speeds, but for the high-speed domain, no improvement was found.

The modeling of variable speed milling is more complex than that of turning, since the speed variation frequency and the tooth passing frequency interact and the resulted system is typically quasi-periodic. Still, there are mathematical techniques to determine approximate dynamic properties. Sastry et al. [21] used Fourier expansion and applied the Floquet theory to derive stability lobe diagrams for face milling. They obtained some improvements for low spindle speeds. Recently, Zatarain et al [22] presented a general method in frequency domain to the problem, and show that variable spindle speed can effectively be used to chatter suppression for low cutting speeds. They used the semi-discretization method and time domain simulations to validate their model, and confirmed their results by experiments.

In this paper, the stability of variable speed milling is analyzed in the high-speed domain, for the spindle speeds corresponding to the first flip (period doubling) and to the first Hopf lobes. Theoretical stability predictions are obtained using the semi-discretization method based on [23], and the results are confirmed by experiments.

2 MODEL OF MILLING PROCESS WITH VARIABLE SPINDLE SPEED

2.1 Variation of the spindle speed

Periodic spindle speed variation is considered in the form $N(t) = N_0 + N_A S(t)$, where N_0 is the mean spindle speed, N_A, is the amplitude of the variation and $S(t) = S(t+T)$ is a T-periodic shape function that varies between -1 and 1. In the literature, mostly sinusoidal, triangular or square-wave modulations are considered. Here, the triangular variation shown in Figure 1 is analyzed. The corresponding shape function reads:

$$S(t) = \begin{cases} 1 - 4\,\mathrm{mod}(t,T)/T & \text{if} \quad 0 < \mathrm{mod}(t,T) \le T/2 \\ -3 + 4\,\mathrm{mod}(t,T)/T & \text{if} \quad T/2 < \mathrm{mod}(t,T) \le T \end{cases} \tag{1}$$

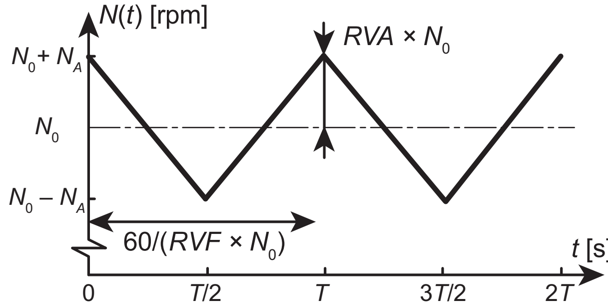

Figure 1: Typical triangular shape variation.

Here, $\mathrm{mod}(t,T)$ denotes the modulo function, for example, $\mathrm{mod}(12, 5) = 2$.

According to the general notation in the corresponding literature, the amplitude and the frequency of the speed variation is normalized by the mean spindle speed N_0 as:

$$RVA = \frac{N_A}{N_0}, \tag{2}$$

$$RVF = \frac{60}{N_0 T} = \frac{60 f}{N_0}. \tag{3}$$

RVA represents the ratio of the amplitude N_A and the mean value N_0. In practical applications, the maximum value for RVA is about 0.3. This represents a variation of 30% of the spindle speed and results in a variation of 30% of the feed by tooth due to the constant feed velocity. RVF is the ratio of the variation frequency f and the average spindle frequency $N_0/60$. The variation frequency f is typically about 1-2 Hz. Using the normalized parameters introduced above, the triangular modulation can be given as:

$$N(t) = \begin{cases} N_0(1 + RVA) - \dfrac{4N_0\,RVA}{T}\,\mathrm{mod}(t,T) & \text{if} \quad 0 < \mathrm{mod}(t,T) \le T/2 \\ N_0(1 - 3RVA) + \dfrac{4N_0\,RVA}{T}\,\mathrm{mod}(t,T) & \text{if} \quad T/2 < \mathrm{mod}(t,T) \le T \end{cases}. \tag{4}$$

The time delay between two subsequent cutting teeth plays a crucial role in the system's dynamics due to the regenerative effect. For a machining process with constant spindle speed N_0, this time delay is constant:

$$\tau_0 = \frac{60}{zN_0}, \tag{5}$$

where z is the number of the teeth of the tool. For variable spindle speed machining, the time delay varies periodically in time according to the spindle speed modulation. The variation of the regenerative delay can be given in the implicit form:

$$\int_{t-\tau(t)}^{t} \frac{N(s)}{60}\,\mathrm{d}s = \frac{1}{z}. \tag{6}$$

For the triangular modulation defined in Eq. (4), this equation gives:

$$\tau(t) = \begin{cases} \tau_0(1-RVA) + \dfrac{4\tau_0 RVA}{T}\operatorname{mod}(t,T) & \text{if} \quad 0 < \operatorname{mod}(t,T) \le T/2 \\ \tau_0(1+3RVA) - \dfrac{4\tau_0 RVA}{T}\operatorname{mod}(t,T) & \text{if} \quad T/2 < \operatorname{mod}(t,T) \le T \end{cases} \tag{7}$$

where τ_0 is the mean time delay.

2.2 The mechanical model

A schematic diagram of the milling process is shown in Figure 2. The structure is assumed to be flexible in the x direction, while the feed is parallel to the y direction. The dynamic model is given by the following equation:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F_x(t), \tag{9}$$

where m is the modal mass, c is the damping, k is the stiffness and $F_x(t)$ is the cutting force in the x direction. According to the linear cutting law, the x component of the force is given by:

$$F_x(t) = A_p \sum_{j=1}^{z} \left(K_R\cos\varphi_j - K_T\sin\varphi_j\right) h_j(t), \tag{10}$$

where A_p is the axial depth of cut and K_T and K_R are the specific tangential and radial cutting coefficients. The chip thickness is expressed by:

$$h_j(t) = g_j(t)\left(f_z\sin\varphi_j + \left(x(t) - x(t-\tau(t))\right)\cos\varphi_j\right), \tag{11}$$

where the function $g_j(t)$ is a unit function, it is equal to 1 when the tooth j is cutting, otherwise it is equal to 0. Here, f_z is the feed per tooth, $x(t)$ is the current position of the tool and $x(t-\tau(t))$ is the position at the previous cut. The regenerative delay $\tau(t)$ is periodic in time due to the spindle speed modulation, as it is given in Eq. (7).

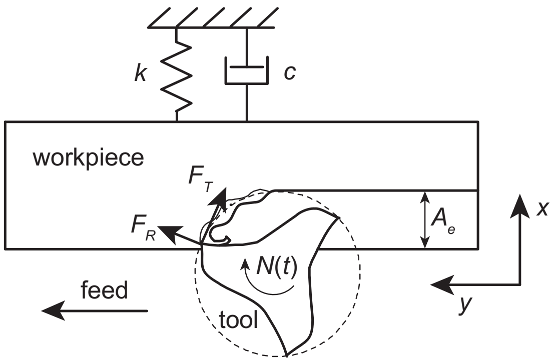

Figure 2: Mechanical model of the milling process with single degree of freedom.

3 THEORETICAL STABILITY PREDICTIONS

Equations (9), (10) and (11) imply the time periodic delay differential equation in the form:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t-\tau(t)), \\ \mathbf{u}(t-\tau(t)) &= \mathbf{C}\mathbf{x}(t-\tau(t)), \end{aligned} \tag{12}$$

where

$$\mathbf{x}(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}, \quad \mathbf{A}(t) = \begin{bmatrix} 0 & 1 \\ v(t) - \dfrac{k}{m} & -\dfrac{c}{m} \end{bmatrix},$$

$$\mathbf{B}(t) = \begin{bmatrix} 0 \\ v(t) \end{bmatrix}, \quad \mathbf{u}(t-\tau(t)) = \left[x(t-\tau(t))\right], \quad \mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

and

$$v(t) = \frac{A_p}{m}\left(\sum_{j=1}^{z} g_j(t)\left(K_R\cos\varphi_j - K_T\sin\varphi_j\right)\cos\varphi_j\right).$$

As it is shown by Eq. (7), the regenerative time delay is periodic at the spindle modulation period T. We assume that the ratio of the modulation period T and the mean time delay τ_0 is a rational number, i.e., $qT = p\tau_0$ with q and p being relative primes. Thus, the system is periodic at the principal period qT, consequently, the Floquet theory of periodic DDEs can be applied. Note that if the ratio of T and τ_0 is not rational, then the system is quasi-periodic and the Floquet theory cannot be used.

The stability is determined using the first-order semi-discretization method according to [24]. The scheme of the approximation is shown in Figure 3. First, the discrete time scale $t_i = i\Delta t$, i=1,2,,... is introduced so that $qT = K\Delta t$ with K being an integer. In the i^{th} discretization interval, the time delay is approximated by its integral average as:

$$\tau_i \approx \frac{1}{\Delta t}\int_{t_i}^{t_{i+1}} \tau(s)\,\mathrm{d}s, \qquad t \in [t_i, t_{i+1}]. \tag{13}$$

Then, the delayed term $\mathbf{u}(t-\tau(t))$ is approximated by the linear function of time as:

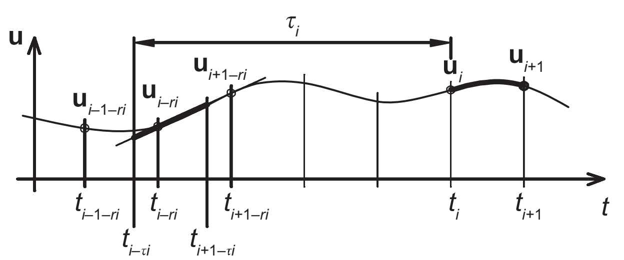

Figure 3: Schematic of the semi-discretization method.

$$\mathbf{u}(t-\tau(t)) \approx \mathbf{u}(t-\tau_i) \approx \frac{t-\tau_i-t_{i-r_i}}{\Delta t}\mathbf{u}_{i+1-r_i} + \frac{t_{i+1-r_i}+\tau_i-t}{\Delta t}\mathbf{u}_{i-r_i}, \quad t\in[t_i,t_{i+1}] \tag{14}$$

where

$$r_i = \mathrm{int}\left(\frac{\tau_i}{\Delta t}+\frac{1}{2}\right), \tag{15}$$

and $\mathbf{u}_i = \mathbf{u}(t_i)$ is used as short notation. Note that $r_i\Delta t$ is a kind of integer approximation of the delay τ_i. Finally, the time periodic functions are approximated by their integral average:

$$\mathbf{A}_i = \frac{1}{\Delta t}\int_{t_i}^{t_{i+1}}\mathbf{A}(s)\,\mathrm{d}s, \quad \mathbf{B}_i = \frac{1}{\Delta t}\int_{t_i}^{t_{i+1}}\mathbf{B}(s)\,\mathrm{d}s. \tag{16}$$

Now, in each interval $t\in[t_i,t_{i+1}]$, the equation of motion (12) is approximated by:

$$\dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\left(\frac{t-\tau_i-t_{i-r_i}}{\Delta t}\mathbf{u}_{i+1-r_i} + \frac{t_{i+1-r_i}+\tau_i-t}{\Delta t}\mathbf{u}_{i-r_i}\right). \tag{17}$$

This system can be considered as an ordinary differential equation with a forcing term, which linearly depends on time. Thus, if $\mathbf{x}_i = \mathbf{x}(t_i)$, $\mathbf{u}_{i+1-r_i} = \mathbf{u}(t_{i+1-r_i})$, $\mathbf{u}_{i-r_i} = \mathbf{u}(t_{i-r_i})$ are given, then the solution over the interval $t\in[t_i,t_{i+1}]$ can be constructed analytically as:

$$\mathbf{x}_{i+1} = \mathbf{x}(t_{i+1}) = \mathbf{P}_i\mathbf{x}_i + \mathbf{R}_{i0}\mathbf{u}_{i-r_i} + \mathbf{R}_{i1}\mathbf{u}_{i+1-r_i}, \tag{18}$$

where

$$\mathbf{P}_i = \exp(\mathbf{A}_i\Delta t),$$

$$\mathbf{R}_{i0} = \left(\mathbf{A}_i^{\,2} + \tfrac{1}{\Delta t}\mathbf{A}_i^{\,-2}\left(\mathbf{I}-\exp(\mathbf{A}_i\Delta t)\right) - \tfrac{\tau_i-(1+r)\Delta t}{\Delta t}\mathbf{A}_i^{\,-1}\left(\mathbf{I}-\exp(\mathbf{A}_i\Delta t)\right)\right)\mathbf{B}_i,$$

$$\mathbf{R}_{i1} = \left(-\mathbf{A}_i^{\,-1} + \tfrac{1}{\Delta t}\mathbf{A}_i^{\,-2}\left(\mathbf{I}-\exp(\mathbf{A}_i\Delta t)\right) - \tfrac{r\Delta t-\tau_i}{\Delta t}\mathbf{A}_i^{\,-1}\left(\mathbf{I}-\exp(\mathbf{A}_i\Delta t)\right)\right)\mathbf{B}_i.$$

Here, **I** denotes the 2x2 unit matrix. This solution can be represented by a discrete map:

$$\mathbf{y}_{i+1} = \mathbf{Q}_i\mathbf{y}_i, \tag{19}$$

with

$$\mathbf{y}_i = \begin{bmatrix}\mathbf{x}_i\\ \mathbf{u}_{i-1}\\ \mathbf{u}_{i-2}\\ \vdots\\ \mathbf{u}_{i-r_{\max}}\end{bmatrix}, \quad \mathbf{Q}_i = \begin{bmatrix}\mathbf{P}_i & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{R}_{i1} & \mathbf{R}_{i0} & \mathbf{0} & \cdots\\ \mathbf{C} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots\\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots\\ \mathbf{0} & \mathbf{0} & \mathbf{I} & & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \\ \vdots & \vdots & & \ddots & & \vdots & \vdots & \vdots & \end{bmatrix}$$

where $\mathbf{R}_{i1}$ and $\mathbf{R}_{i0}$ are in the $(r_i\text{-}1)^{\text{th}}$ and the r_i^{th} columns and $r_{\max} = \max(r_0, r_1, \ldots r_{K-1})$. Note, that in this case, the elements $\mathbf{u}_i$ are 1×1 matrices, and the corresponding 1×1 unit matrices **I** below the diagonal are in fact the scalar unit 1.

The approximate Floquet transition matrix can be given after computing matrix $\mathbf{Q}_i$ in K succeeding discretization intervals:

$$\boldsymbol{\Phi} = \mathbf{Q}_{K-1}\mathbf{Q}_{K-2}\ldots\mathbf{Q}_0. \tag{20}$$

If the eigenvalues of $\boldsymbol{\Phi}$ are in modulus less than 1, then the process is stable. Stability lobes can be constructed by scanning the cutting conditions (spindle speed and axial depth of cut) for a couple of (*RVA*, *RVF*) parameters. An example can be seen in Figure 4.

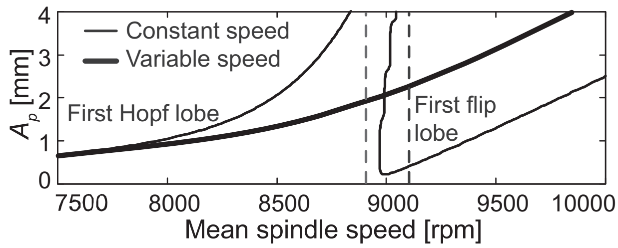

Figure 4: Stability diagrams for constant and for variable speed milling with *RVA* = 0.3 and *RVF* = 0.003

4 SELECTION OF THE OPTIMAL PARAMETERS

Stability of variable speed machining is very sensitive to the choice of the frequency and amplitude parameters. In order to find the optimal modulation, different combinations of frequencies and amplitudes should be analyzed. Here, the effectiveness of the spindle speed variation is investigated in the area of the 1st flip lobe (N_0 = 9100 rpm, blue dashed line in Figure 4) and also in the area of the 1st Hopf lobe (N_0 = 8900 rpm, red dashed line in Figure 4). For these spindle speeds, the critical depths of cut were determined for several modulation amplitudes (*RVA*) and frequencies using the semi-discretization method. The results for average spindle speed of 9100 rpm are presented in Figure 5 in contour plot form. For constant speed, the critical depth of cut is 0.5 mm. For variable spindle speed, the critical depth of cut A_p is always larger than 0.5 mm for any *RVA* and *RVF* values. For some domains, even A_p = 2.4 mm can be achieved that corresponds to 380% increase in the depth of cut. Figure 5 also shows that large frequency of speed variation coupled with small amplitude does not yield any gain in the depth of cut. The most effective parameter is the amplitude variation, while the frequency does not have a significant effect on the stability within the range of 0.5-4 Hz. The selection of the frequency and the amplitude variation is limited by the spindle's dynamics that is also denoted in the frequency-amplitude diagram in Figure 5. Considering this limit, the optimal choice is to apply low frequency of modulation with large amplitude. Such cases are denoted by points A and C.

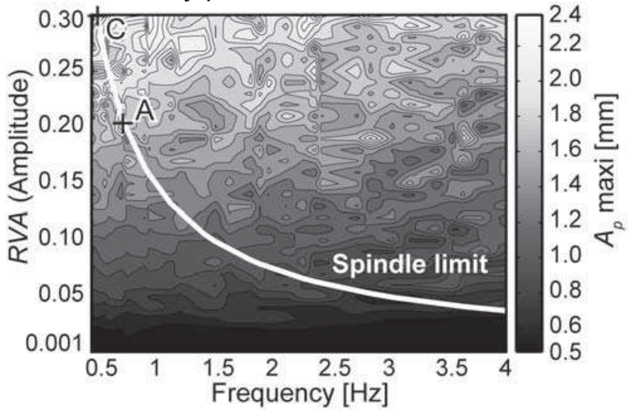

Figure 5: Parametric study for N_0 = 9100 rpm and A_e = 2 mm.

Similar plot was determined spindle speed 8900 rpm shown in Figure 6. In this case the critical depth of cut for constant speed machining was 5 mm. It was found that spindle speed variation always result in a depth of cut smaller than 5 mm, thus in this case, the application of varying spindle speed is not useful.

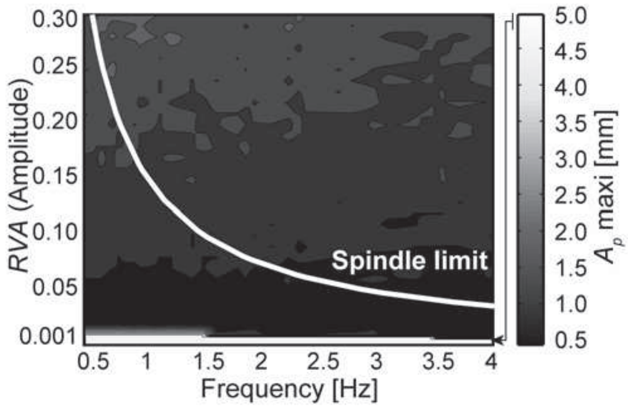

Figure 6: Parametric study for N_0 = 8900 rpm and A_e = 2 mm.

Based on the above numerical studies, it can be concluded that the efficiency of spindle speed variation in high speed milling is diverse for different spindle speeds. For the area of the 1st flip lobe, the critical depth of cut can essentially be increased as it is shown in Figure 5. However, for the area of the 1st Hopf lobe, no significant gains in the depth of cut can be achieved by spindle speed variation. It is found, furthermore, that the improvements depend mostly on the amplitude of the speed variation, and the dependence on the frequency is modest.

5 EXPERIMENTAL WORK

The machining tests were carried out on a high-speed milling center (Huron, KX10). The average feed per tooth was 0.1 mm/tooth. The tool was an inserted mill with 3 teeth, D = 25 mm diameter without helix angle. The spindle speed variation was implemented by a sub-program using a synchronous function (Siemens 840D). In compliance with the dynamics of the spindle, the difference between the input and the measured spindle speed trajectory was less than 0.5%, see Figure 7. According to Siemens, the spindle speed variation has no negative effect on the spindle life.

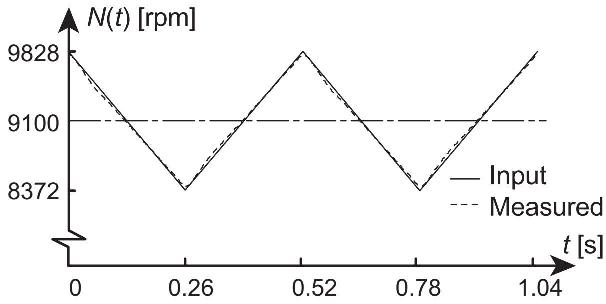

Figure 7. Comparison between input and measured spindle speed trajectory, for N_0 = 9100 rpm ; RVA = 0.08 ; RVF = 0.0125 (f = 1.9 Hz).

The setup of the milling tests can be seen in Figure 8. A flexure was used to provide a single degree of freedom system that is compliant in the x direction (perpendicular to the feed). The tool is considered to be rigid compared to the flexure. An aluminium (2017A) part was down-milled with a radial depth of cut A_e = 2 mm, thus the radial immersion ratio was A_e/D = 0.08. The length of the workpiece was 90 mm, the operation time was approximately 2 s at spindle speed 9100 rpm. The vibrations of the part were measured by a laser velocimeter (Ometron, VH300+). A filtering followed by a numerical integration was used to extract displacement of the part.

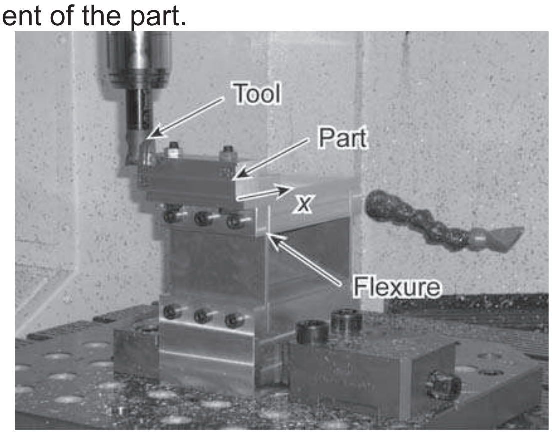

Figure 8. Experimental setup.

The dynamic characteristics of the system were determined by hammer impact test. The modal parameters and the cutting force coefficients are collected in Table 1. The cutting force coefficients were determined in previous work [10].

m [kg]	f_0 [Hz]	ξ [%]	K_T [MPa]	K_R [MPa]
1.637	222.5	0.50	700	140

Table 1. Modal parameters and cutting force coefficients.

5.1 Constant spindle speed tests

First, a series of tests at a constant speed has been conducted in order to verify the model. The results are shown in Figure 9. Stable cutting tests are denoted by circle while unstable tests by crosses. The predicted behavior of the system agrees well with the experiments. The zone of period doubling chatter at the 1st flip lobe is also explored using a finer resolution of the spindle speed.

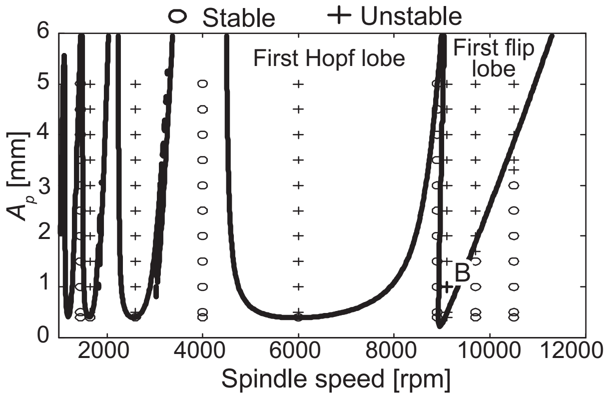

Figure 9: Experimental test at constant spindle speed.

5.2 Stabilization via spindle speed variation

In this section, chatter suppression by spindle speed variation is presented by an example. Consider the machining process with spindle speed 9100 rpm and depth of cut 1 mm. For constant spindle speed, this process is unstable (see Figure 4). Spindle speed variation is applied according to point A in Figure 5. The corresponding parameters are RVA = 0.2, RVF = 0.0046875 (f = 0.71 Hz). Based on the theoretical predictions in Figure 5, the critical depth of cut is about 2 mm, i.e., the system with variable spindle speed is predicted to be stable. Figure 10 presents the spindle speed, the displacement history and the surface roughness obtained by the cutting tests.

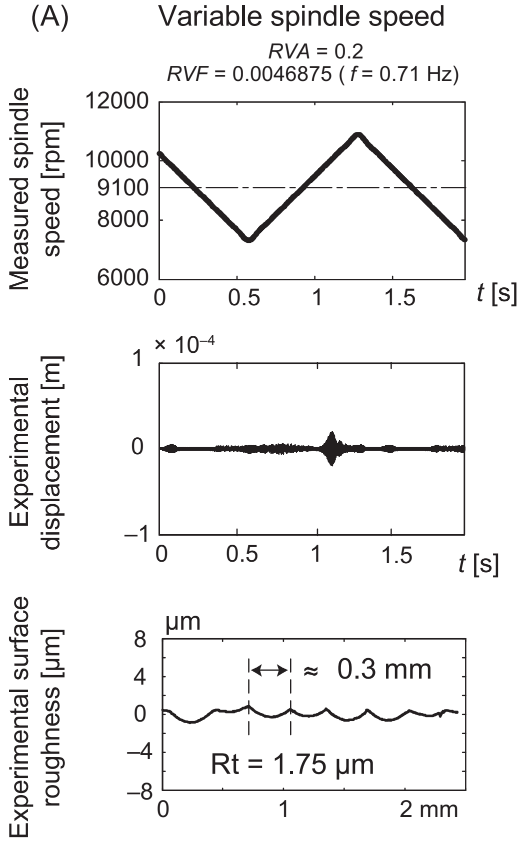

Figure 10: Chatter suppression by spindle speed variation for A_p = 1 mm and N_0 = 9100 rpm.

For an ideally symmetric tool, the pitch of the machined profile is equal to the feed per tooth. However, if the tool has a runout larger than the roughness of the surface, than it leaves only one mark by revolution. The tool used in the tests had a runout of 10 μm, the feed per tooth was 0.1mm and the tool had 3 teeth, thus the pitch of the machined profile is expected to be approximately 0.3 mm for stable machining. (In fact, the pitch slightly varies around 0.3 mm, since the constant feed velocity and the variable spindle speed produces a varying feed per tooth.)

During the tests with variable spindle speed, no chatter was observed. The amplitude of the vibrations was less than 0.01 mm, the roughness was 1.75 μm and the pitch of the machined profile was 0.3 mm as it can be seen in Figure 10. These all refer to a stable cutting process.

The results for constant spindle speed are shown in Figure 11. In this case chatter was clearly identified. The amplitude of the vibrations was about 0.07 mm, the roughness was 3.7 μm, and the pitch of the machined profile was 0.6 mm that refers to the period doubling chatter (note that these cutting parameters are in the 1st flip zone).

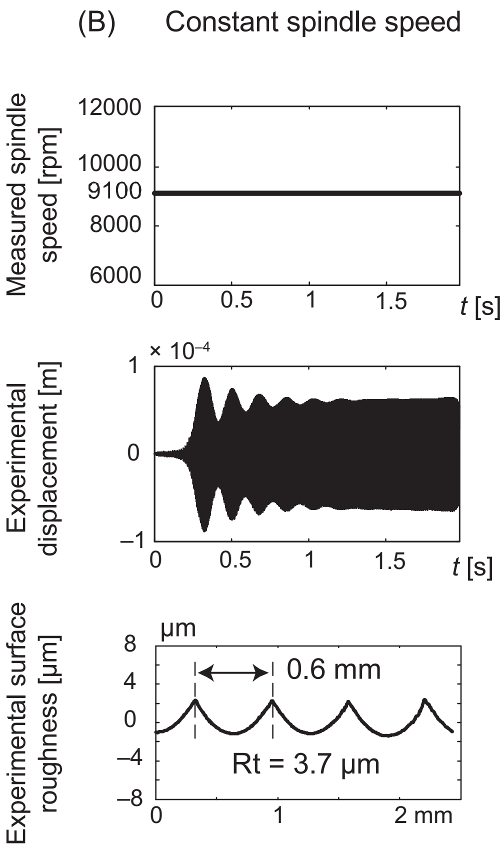

Figure 11. Constant spindle speed machining for A_p = 1 mm and N_0 = 9100 rpm.

6 CONCLUSIONS

Variable spindle speed machining has been studied for high-speed milling at around the 1st Hopf and the 1st flip lobes. Stability properties were predicted using the semi-discretization method, and confirmed by time-domain simulations. Different combinations of the amplitude and the frequency of the speed modulation were analyzed in order to find the optimal technique to suppress chatter. It was found that the stability properties can always be improved (i.e., the critical depth of cut can always be increased) by spindle speed variation within the unstable domain of the 1st flip lobe, while there are some spindle speeds, where the spindle speed variation does not provide any essential gain. It was also shown that the amplitude has a stronger effect on the stability of the process than the frequency.

Cutting tests were performed for certain spindle speeds in the flip domain in order to verify the theoretical predictions. The stabilizing effect of spindle speed variation was clearly verified experimentally, a period doubling chatter was suppressed by applying a proper spindle speed variation.

The efficiency of spindle speed variation can be increased by spindles that are capable to provide large acceleration. This would allow the use of larger range of amplitudes and frequencies. Furthermore, the application of large modulation frequencies would decrease the principal period of the system that would be beneficial for the suppression of transient vibrations.

Finally, it should be noted, that practically, it might be easier to change the spindle speed to stable parameter domains rather than applying spindle speed variation. For instance, in the current study, application of constant speeds 8900 rpm or 11000 rpm both result in a stable machining process, as well, even for depths of cut 4-5 mm. However, the structure of the stability diagram is usually unknown for the machinist, and to find a stable "window" in the stability lobe diagram is not a trivial task. For a complex part with multiple modes, the lobe diagram is more intricate than that of a single degree-of-freedom system. In these cases, the application of spindle speed variation is a more practical than searching for stable spindle speeds. Furthermore, the spindle speed is often limited by other technological factors. In this context, spindle speed variation is an extra tool in addition to the traditional methods that can be used in the complex optimization of machining processes.

7 ACKNOWLEDGMENTS

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