Probabilistic Assessment of the Efficiency of Algorithms for Crack Detection on Instrumented RC Beams: a non-Model Based Method

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ABSTRACT

Damage detection of instrumented structures has known large developments during the last decades. Some of the approaches are called model-based as they lie on a mechanical model of the structure and aim to model and identify the crack characteristics from monitoring. The main disadvantage of this approach for real structures is to model other effects that may change the response of the sensor: temperature, traffic loading, etc. Other approaches are called non-model based methods: they lie on the analysis of changes in the response only. Their main disadvantage is that the identification of the crack characteristics is unavailable. In parallel with the diffusion of monitoring systems, the development of decision support systems (DSS) has been started, that utilize the results of the monitoring data interpretation process for supporting decisions concerning maintenance/rehabilitation planning. The overall process, from data interpretation and damage identification to the definition of the lifetime or fragility curves and finally to decision models is however affected by several uncertainties. The talk will focus on the uncertainties of the damage identification process suggesting a probabilistic measurement of the change in the response through an approach largely used for Non Destructive Testing calibration. The reliability estimate of the damage identification algorithms will be evaluated defining the probability of detection (PoD), the probability of false alarms (PFA) and a receiver operating characteristic (ROC) curve. For a given detection threshold, the couple (PoD, PFA) defines the efficiency of a detection algorithm. The objective is to maximize the probability of detection and minimize the probability of false alarm.

The performance of the Proper Orthogonal Decomposition in detecting and localizing damage will be presented, regarding the application to long-term static monitoring data resulting from an experiment conducted in real environmental conditions on two pre-stressed concrete beam specimens. These ROC curves can be obtained by considering two or more algorithms and the same defect range, one technique and two defect ranges, one technique with two settings and the same defect range, or one technique applied in various conditions (even if the testing procedure is rigorously followed during inspection). In the following we will consider two settings (rough data and pre-processed data) and the same defect. The paper will also

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present a combination of several sensors measurements to illustrate the gain obtained from a redundant monitoring.

Keywords: *Distribute fiber optic sensors, reinforced concrete, crack, Probability of Detection, Probability of False Alarm.*

1 INTRODUCTION

Safety and usability of existing concrete structures actually depend on the level of degradation and obsolescence due to ageing. The time-dependent reliability problem for ageing concrete structures was discussed by several authors, among which [1], [2] and [3] have been pioneering the subject.

Reliability analysis in structural engineering has to enable quantification of uncertainties associated with loading, materials, geometry, deterioration, modelling and other factors. These are integrated into a method that estimates the probability of reaching the specified performance level during the service life of a structure [4]. Lifetime functions for concrete structures can be constructed from theoretical/experimental degradation models and, if a Structural Health Monitoring system is installed on the structure, at any required time, the actual state of degradation can also be determined by applying a damage identification process, allowing updating of the lifetime functions [5]. For example, the use of monitored extreme data allows the reduction of uncertainties associated with numerical models, and the validation and updating of existing prediction models [6]. For complex structures in particular, SHM has been proved to be the only way to understand both complex interaction mechanisms, as soil-structure interaction, and in-service structural behaviour influenced by building/loading conditions and settlements that are expected to separate the current behaviour from the theoretical one [7][8]. The overall process, from data interpretation and damage identification to the definition of the life-cycle curves and finally to decision models is anyway affected by several uncertainties, both stochastic and epistemic.

The SHM efficiency depends on the ability to detect the structural changes under environmental and operational conditions [9]. A lot of damage detection methods have been tested in the last years, on static [10] and dynamic data [11], and on laboratory [12] and in-field experiments [13][14]. Despite of the above interest that has been largely expressed by means of research papers in conferences and scientific journals, very few practical examples can be reported to date, the main reasons being the lack of extensive field evidence in the disclosure of structural defects and the inability to interpret the data obtained from hundreds of sensors in a timely manner. Robust and reliable methods capable of detecting, locating and estimating damage whilst being insensitive to changes in environmental and operating conditions have yet to be agreed upon.

With reference to a real case of permanent static heath monitoring systems [15], the paper is aimed at discussing the performance of the Proper Orthogonal Decomposition as a damage detection algorithm [16]. The damage indicators extracted from the proposed algorithm are used to set a reference period and a threshold value associated to damage presence. The uncertainties affecting the damage identification process on the assessment of the structural health state will be taken into account, in particular uncertainties due to environmental changes.

First, theoretical aspects from detection theory are reminded. Then, based on the characteristics of real monitoring data, the experimental approach is presented and the performance estimate of damage detection algorithms (DDA) for a given detection threshold is discussed through the Receiver Operating Characteristic (ROC) curve [17]. Looking for the best detection performances, the probability of detection should always take larger values than the probability of false alarm (low noise sensitivity).

In the paper, special emphasis is given to the influence of the environmental conditions on the probability of false alarm, because this latter depends on the noise level and the chosen threshold only. On the contrary, the probability of detection depends on the damage extension. The difficulties in obtaining a good performance curve in presence of strong temperature variations (noise) and small levels of damage is also discussed and the possibility of improvement of the detection process through post-processing of acquired data is illustrated. Data from a 2-year monitoring program on pre-stressed beams is considered for illustration and different ROC curves are deduced as a function of the threshold assumed for damage identification. The discussion is based on the availability of a reference period in which the structure can be considered as undamaged.

2 PROBABILISTIC MODELING OF DETECTION

The objective of the following sections is to present the expansion of the probabilistic modelling of inspection results through NDT tools to the probabilistic modelling of damage detection from data processing algorithms.

2.1 Probability of Detection and Probability of False Alarm

The most common concept which characterizes the performance of inspection tools is the probability of detection PoD. Let a_d be the minimal defect size, called the detection threshold, under which it is assumed that no detection is possible. PoD is defined as (Eq. (1)):

$$PoD = P\left(\hat{D} \geq a_d\right) \tag{1}$$

where $P()$ represents a probability measure, $\hat{D}$ is the variable that represents the measured defect size $\hat{d}$ (response level of NDT tool i.e. 'signal+noise'). The real defect size (i.e. the real signal without noise) is D.

Assuming that the probability density functions of noise and signal amplitude are known, after fitting empirical distribution for instance, PoD and the probability of false alarm PFA have the following expressions (Eq. (2) and (3)):

$$PoD = \int_{a_d}^{+\infty} f_{\hat{D}}\left(\hat{d}\right)\partial\hat{d} \tag{2}$$

$$PFA = \int_{a_d}^{+\infty} f_{\Lambda}\left(\eta\right)\partial\eta \tag{3}$$

where η is the noise, $f_{\hat{D}}$ and f_{Λ} are respectively the probability density functions of the 'signal+noise' $\hat{D}$ (or measured defect) and the 'noise' Λ. In the following noise Λ and signal D are considered as independent random variables.

Thus, PoD is a function of the detection threshold, the measured defect size and the noise while PFA depends on the detection threshold and noise only [17]. Noise is dependent on the decision-chain "physical measurement-decision on defect measurement transfer of information", the conditions of inspection (harsh environment, surface quality, electronic noise) and the complexity of testing procedure (accessibility, mounting of the device). Figure 1 illustrates the Probability Density Function (PDF) and the area to be computed for the evaluation of PoD and PFA for a given detection threshold in the case where 'signal+noise' $\hat{D}$ and 'noise' Λ are normally distributed.

2.2 Receiver Operating Characteristic (ROC) curve

The ROC curve links the Probability of Detection and the Probability of False Alarm. For a given detection threshold, the pair (PFA, PoD) defines NDT performance. This pair can be considered as coordinates of a point in R^2 (square integrable space of real numbers) with axes representing PFA and PoD. Let us consider that a_d takes values in the range $]-\infty;+\infty[$, these points belong to a curve called Receiver Operating Characteristic (ROC) which is a parametric curve with parameter a_d and defined by Eq. (2) and (3).

The ROC curve (ROC3) plotted on Figure 2 is computed with the PDF presented on Figure 1 corresponding to normal distributions.

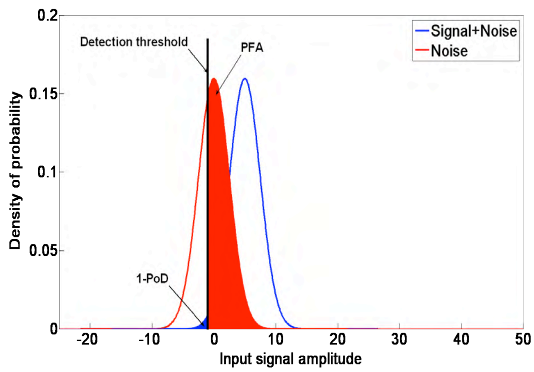

Figure 1 - Illustration of PoD and PFA (signal+noise and noise normally distributed) for detection threshold a_d.

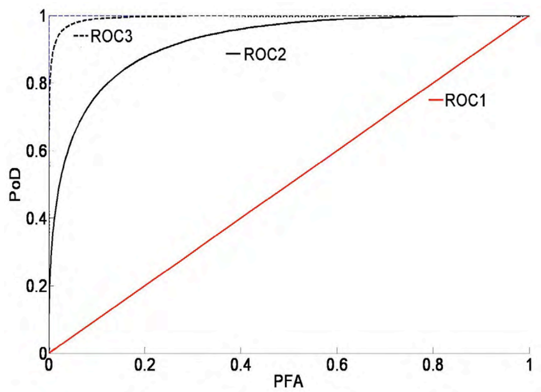

Figure 2 - Example of ROC curves with several NDT performances.

The ROC curve is a fundamental characteristic of the NDT tool performance for a given PDF of a defect or a defect range . Perfect tool is represented by a ROC curve reduced to a single point whose coordinates are: (PFA, PoD) = [0, 1]. A ROC curve represents a NDT tool performance facing a given PDF of a defect or a defect range [18].

Figure 2 presents 3 theoretical ROC curves, each one corresponding to a different NDT tool performance. The worst curve is ROC1, meaning that noise can be easily detected as a defect even if nothing is to be detected. This finally leads to a high number of false alarms. As a result, overall performances are poor. In contrast, the best plotted ROC curve is ROC3, which differs considerably from the previous curve. The probability of detection reaches values near to 1, with small probabilities of false alarms for high values of PoD. Overall performances are very good.

These ROC curves can be obtained by considering two techniques and the same defect range, one technique and two defect ranges, one technique with two settings and the same defect range, or one technique applied in various conditions (even if the testing procedure is rigorously followed during inspection). In the following two settings and the same defect will be considered.

A simple geometric characterization of ROC curves is the distance between the curve and the Best Performance Point (BPP) of coordinates (PFA=0, PoD=1) [19]. By definition, the bigger the distance, the worse is the performance. The point on the ROC curve corresponding to the lowest distance between BPP and the curve is called the performance point of the NDT tool (NDT-BPP). This distance (Euclidean measure) can thus be considered as a measure of performance. This paper defines a curve characterization by using the polar coordinates of the NDT-BPP. The NDT-BPP polar coordinates are defined by:

- the radius α_{NDT} equals the performance index (NDT-PI) (distance between the best performance point and the ROC curve) ([17][19][20]);
- the δ_{NDT} is the angle between axis (PFA=0) and the line (BPP, NDT-BPP).

It has been shown in [21] that α_{NDT} is essential to provide complete risk analysis, including consequence assessment after inspection. However, such a study is beyond the scope of this particular study so this parameter will not be analyzed here. Assessment of PoD and PFA from the knowledge of detection threshold can be directly deduced from inter-calibration of NDT tools from statistical analysis of inspection results ([21][22]). Generally these projects are expensive, and consequently, it is sometimes necessary to choose another approach. Calculation of PFA and PoD thereby results from probabilistic modeling of the 'noise' and 'signal+noise' PDF ([17][19]). Note that other authors [23] have suggested other concepts like Probability of False Indication (PFI). The main disadvantage is that Probability of False Alarm and Probability of No Detection (1-PoD) are mixed and that the size of the defect and the modeling of the noise are no more present.

2.3 Statistically based ROC building for algorithm performance assessment

Here the objective is to find the best parameters for the algorithm that lead to a good assessment of defect. ROC curves are built from statistics and then the best set of parameters that lead to the detection threshold that minimizes δ_{NDT} are deduced. Thus PoD and PFA are computed from Eq. (4) and (5) [11]:

$$PoD \approx \frac{Card(A)}{n_m} \quad with\ A = \left\{ i \in \Im;\ \hat{f}_{p,j} > a_d \right\} \tag{4}$$

$$PFA \approx \frac{Card(B)}{n_m} \quad with\ B = \left\{ i \in \Im; \eta_j > a_d \right\} \tag{5}$$

Where *Card(.)* indicates the cardinal of a particular set and where $\Im = \{1, \ldots, n_m\}$ and n_m denotes the number of measurements.

3 PRACTICAL ASSESSMENT USING MONITORING DATA

As briefly mentioned in the introduction, despite of the great number of damage identification approaches that have been proposed in the literature, there is an increasing need to assess the performance and the validity of the selected algorithms with a comparative tool. The optimal damage identification approach should be able to detect the smaller possible damage at the right place and at the right time.

The probability of detection and the probability of false alarm are both needed to characterize, even partially, the results of a DDA. All algorithms have limitations and, in particular in complex environment and harsh conditions, their capabilities and abilities to well perform are different from those given by numerical simulations and laboratories sets. This leads to lower performances than theoretically expected.

In the following, definitions taken from the assessment of non-destructive testing performance are used. A parallel approach is used to evaluate the performance of a damage detection algorithm already applied by the Authors ([15][16]) on numerical simulation and experimental data. As previously mentioned, the inspection results from a NDT are herein substituted by the damage identification results from the Proper Orthogonal Decomposition.

More generally in this framework, the variables $\hat{D}$ and a_d in Eq. (1) can represent the observed current value and the threshold value of one or more features extracted from the measurements by the particular damage identification algorithm. In this case, the threshold value a_d should be defined observing the variation of the feature/s for a reference period during which the structure is supposed to be undamaged. The length of the reference period depends on the available information and on the periodicity of the time histories as shown in [24].

As the definition of the threshold value in the case of damage detection algorithm is not easy to achieve, a parametric study using a vector $\boldsymbol{c}$ of threshold values is suggested. The threshold can be for example defined using an adequate confidence interval assessed from the observations of the reference period. The best threshold value has to be set for each tested DDA.

When assessing the ROC curve using experimental data, discrete values for PFA and PoD are obtained for given conditions.

3.1 Presentation of the experiment and data analysis

An experimental monitoring program, carried out by the Authors in cooperation with Autostrade per l'Italia S.p.A., EPFL and SMARTEC SA, has been performed to validate different damage identification algorithms, previously validated on numerically simulated experiments, that can be applied to the SHM of reinforced and pre-stressed concrete beams under static continuous monitoring programs. The experiment has been conducted for more than two years and consists in the continuous monitoring of strains and temperatures in two pre-stressed concrete beam specimens, placed outdoor under environmental conditions and one of which has been subjected to increasing artificial damages. The strain time histories have been detected by means of fibre optic SOFO sensors (eight installed on the damage beam, four on the undamaged one). The details of the experimental program, the results of some applied damage detection algorithms and the method proposed to remove the temperature effects are presented in [25].

In the present paper the performance of the Proper Orthogonal Decomposition as a damage detection method is investigated. More details on the algorithm can be found in [16]. In this study POD has been proposed to extract a spatial and temporal correlation between the installed sensors, related in a certain way to the normal modes contained in the static structural response. The measurements have to be arranged to form the measurement matrix: each column represents a displacement history at a particular sensor location and each row represents the spatial distribution of the response at a given time instant. After subtracting the mean displacements of each sensor location, the covariance matrix has to be computed and the relative proper orthogonal modes can be obtained. The eigenvalues and the orthogonal eigenvectors are calculated from the covariance matrix using a window containing at each step all previous measurements.

3.2 Assessment of ROC points

The eigenvectors extracted by the Proper Orthogonal Decomposition are used as damage features in the analysis of PoD and PFA. For each structural static mode, the time variation of the relative eigenvector at each sensor location is available. In this way, the method allows to follow the time variations of the eight extracted eigenvectors, but gives also a spatial information following each eigenvector at each sensor location. As a consequence, 64 time trajectories are available to assess the ROC point.

The extracted eigenvectors need a certain period of time to get a stationary behavior to be assumed as reference state. As damages have been introduced too early during the monitoring, they are mixed to the non-stationary initial trend of the eigenvectors; for this reason, three months at the end of the monitoring have been assumed as the reference undamaged period to define the damage detectable threshold. Three thresholds have been defined using the standard deviation of measurements of the reference state: the first one corresponds to a confidence interval of 67% (mean +/- standard deviation), the second one to the 95% (mean +/- 2*standard deviation) and the third one to the 99% (mean +/- 3*standard deviation).

3.3 Results on noisy and pre-processed data

The computation is initially performed on rough data. The extracted eigenvectors contain the structural response and also all the environmental information. A vector of damage threshold is defined for each structural mode at each sensor location. A time window of 5 and 15 days respectively is used to discretize the monitoring period in N intervals, each one containing or not the artificial damage. The PoD and the PFA are evaluated as described in section 2.3 giving a maximum value for the PoD of 0.82 and a minimum value for the PFA of 0.18 for the first mode. No significant differences are noticed in results obtained with the two time windows. The ROC points are then obtained for each eigenvectors and plotted for the three investigated thresholds. Considering the distance α_{NDT} between the experimental points and the best performance point with coordinate (0, 1), it is possible to have an evaluation of the optimal efficiency of the DDA under specific conditions. Figure 3 shows that the higher damage threshold leads to a shorter distance α_{NDT}. Similar results are obtained for the other modes (Figure 4).

The same procedure is performed on pre-processed filtered data to remove the effect of temperature variations, as presented in [25]. In this case, the PoD and the PFA give a maximum value for the PoD of 0.92 (corresponding to +12%) and a minimum value for the PFA of 0.08 (corresponding to -55%). A better performance of the DDA on pre-processed data is evident. The ROC points are finally obtained for each eigenvectors and plotted for the three investigated thresholds. Figure 5 shows again that the higher damage threshold leads to a shorter distance α_{NDT} and that a better efficiency of the DDA is obtained. Similar results are obtained for the other modes (Figure 6).

As confirmed from the previous data interpretation, the performance of the Proper Orthogonal Decomposition improves after the data pre-processing to remove the noise due to temperature effect. A further improvement can be obtained computing the PoD and the PFA using a combination of different eigenvectors. Plots in Figures 7 and 8 show the results using temptative combinations. It can be observed that the best results are obtained combining the first odd modes, 1 and 5 or 1, 3 and 5.

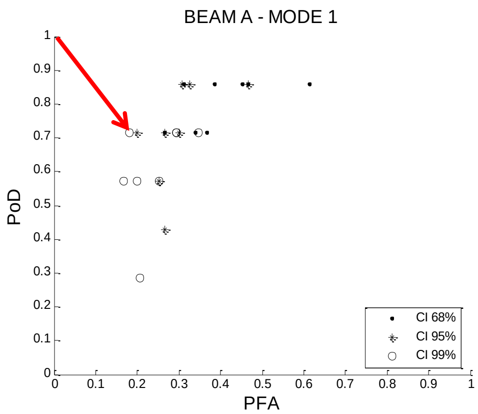

Figure 3 - ROC points from experimental data. Mode 1 at each sensor location (from 1 to 8) corresponding to different damage thresholds.

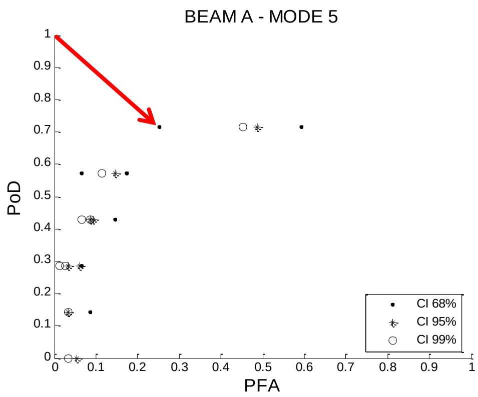

Figure 4 - ROC points from experimental data. Mode 5 at each sensor location (from 1 to 8) corresponding to different damage thresholds.

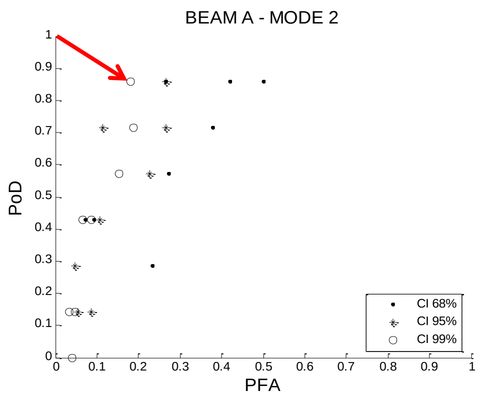

Figure 5 - ROC points from experimental data after pre-processing. Mode 2 at each sensor location (from 1 to 8) corresponding to different damage thresholds.

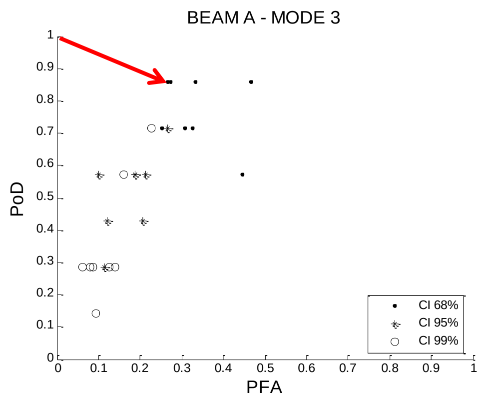

Figure 6 - ROC points from experimental data after pre-processing. Mode 3 at each sensor location (from 1 to 8) corresponding to different damage thresholds.

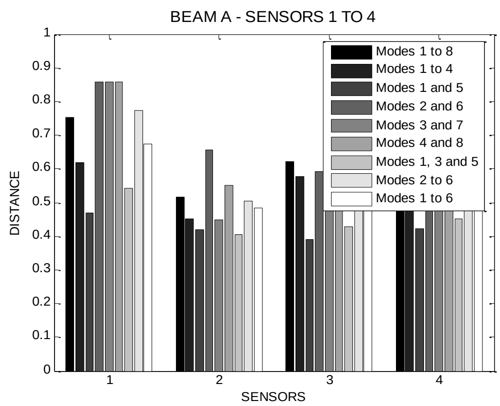

Figure 7 – Distance αNDT for different combinations of eigenvectors (modes) at each sensor location (from 1 to 4).

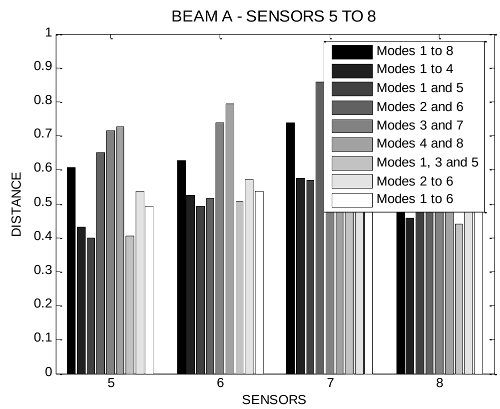

Figure 8 – Distance αNDT for different combinations of eigenvectors (modes) at each sensor location (from 5 to 8).

4 CONCLUSION

The paper has proposed a probabilistic modelling of damage detection from data processing algorithms based on existing probabilistic modelling of NDT techniques. The method has shown to be promising and appropriate when a significant number of sensors are available. The negative influence of the noise due to environmental variations has been shown by comparing the results of the method applied on rough and on filtered data. Significant improvements have been obtained after the removal of the temperature effect on the strain time histories, confirming that deeper studies have to be done for data pre-processing.

The parametric study on the damage level threshold has shown that the higher threshold corresponding to a confidence interval of 99% gives the best performance of the POD algorithm in damage detection. Unfortunately this result has not been observed for all modes, so that a general rule is still impossible to define.

The study of the combination of different modes improves the performance of the algorithm but the results could be significantly improved though the combination of the modes with the higher correlation coefficients. It has to be investigated yet if the combined modes are associated to the same level of the fixed damage threshold.

Interesting results are expected from the comparison of different damage detection algorithms on the same data set.

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