



Zigzag instability in a stratified fluid: a direct transfer of energy

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Nonlinear evolution of the zigzag instability in a stratified fluid

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Abstract :

In a strongly stratified fluid, a columnar counter-rotating vortex pair is subject to the zigzag instability which bends the vortices and ultimately produces layers. We have investigated the nonlinear evolution of this linear instability by means of DNS. We show that the instability grows exponentially without nonlinear saturation and therefore produces rapidly intense vertical shear. We show that this growth is only stopped when vertical viscous effects become important and that it occurs when $F_h^2 Re = O(1)$ where F_h is the horizontal Froude number and Re the Reynolds number. Energy is then rapidly dissipated through viscous effects. We also show that for sufficiently high initial values of $F_h^2 Re$, the intense vertical shear created by the initial zigzag instability is not directly dissipated through viscous effects but first leads to Kelvin-Helmholtz instabilities. This makes the flow turbulent and again rapidly dissipates energy. In both cases, this means that the zigzag instability is a mechanism capable of directly transferring the energy from large scales to small vertical scales where it is dissipated.

Key-words :

stratified flow ; instability ; energy transfer

1 Introduction

In the atmosphere and oceans, flows are usually strongly affected by stable stratification when gravity acts as a restoring force and inhibits vertical displacements. A ubiquitous feature of these flows observed in laboratory or numerical simulations (Fincham *et al.*, 2000; Praud *et al.*, 2005) is that these flows are predominantly horizontal but develop a strong vertical variability. Presently the vertical scaling of these flows and the corresponding energy transfers between vertical scales largely remain open issues. To address this problem, we study numerically a simplified flow with no initial vertical scale imposed and we let vertical scales develop. The flow of interest is initially a two-dimensional pair of counter-rotating vortices. We already now that in a strongly stratified fluid this pair of vortices is subject to a linear 'zigzag' instability that bends the vortices. We present here new results about the nonlinear evolution of this flow.

2 The numerical method

We study numerically the nonlinear evolution of a pair of counter-rotating vortices by using a pseudo-spectral solver of the non-linear incompressible Navier-Stokes equations under the Boussinesq approximation. We already know that the pair of counter-rotating vortices is first bended by the linear zigzag instability, and so the velocity field is initialized as $U_0 = U_{2D}(x, y) + U_p(x, y, z)$ where U_{2D} is the two-dimensional flow corresponding to the unperturbed pair of counter-rotating vortices and U_p is a three-dimensional perturbation of small amplitude corresponding to the most unstable linear mode of the zigzag instability. This form of three-dimensional perturbation was directly added to rapidly destabilize the initial flow and

so to save computational time. For the same reason, we restrict the vertical size of the computational domain to the most unstable wavelength L_z of the zigzag instability. Because we are mainly interested in the subsequent nonlinear evolution of the flow, these simplifications have no impact. Various horizontal Froude numbers F_h and Reynolds numbers Re have been simulated and a large number of collocation points up to $768 \times 768 \times 192$ was used to solve the small vertical scales that appear.

3 Results

3.1 Exponential growth

In a first step, the pair of counter-rotating vortices is subject to the linear zigzag instability that bends the vortices as represented in figure 1. The development of the zigzag instability produces vertical shear that we can follow through the creation of horizontal enstrophy Z_h . The results are presented in figure 2 and show that the instability first grows exponentially without nonlinear saturation. In this first step, viscous forces are unimportant and the vertical lengthscale L_z , which corresponds the most unstable wavelength of the zigzag instability, scales like U/N with U a characteristic horizontal velocity and N the Brunt-Väisälä frequency. These observations are in full agreement with the linear study of the zigzag instability by Billant & Chomaz (2000).

3.2 Viscous dissipation

Because of its development, the instability therefore produces rapidly intense vertical shear. This growth is only stopped (see again figure 2) when vertical viscous effects become important, which occurs when $F_h^2 Re = O(1)$. Energy is then rapidly dissipated through viscous effects.

We could also show that for sufficiently high initial values of $F_h^2 Re$, the intense vertical shear created by the initial zigzag instability is not directly dissipated through viscous effects but first leads to Kelvin-Helmholtz instabilities as shown in figure 3. This makes the flow turbulent and again rapidly dissipates energy.

In both cases, this means that the zigzag instability is a mechanism capable of directly transferring the energy from large scales to small vertical scales where it is dissipated.

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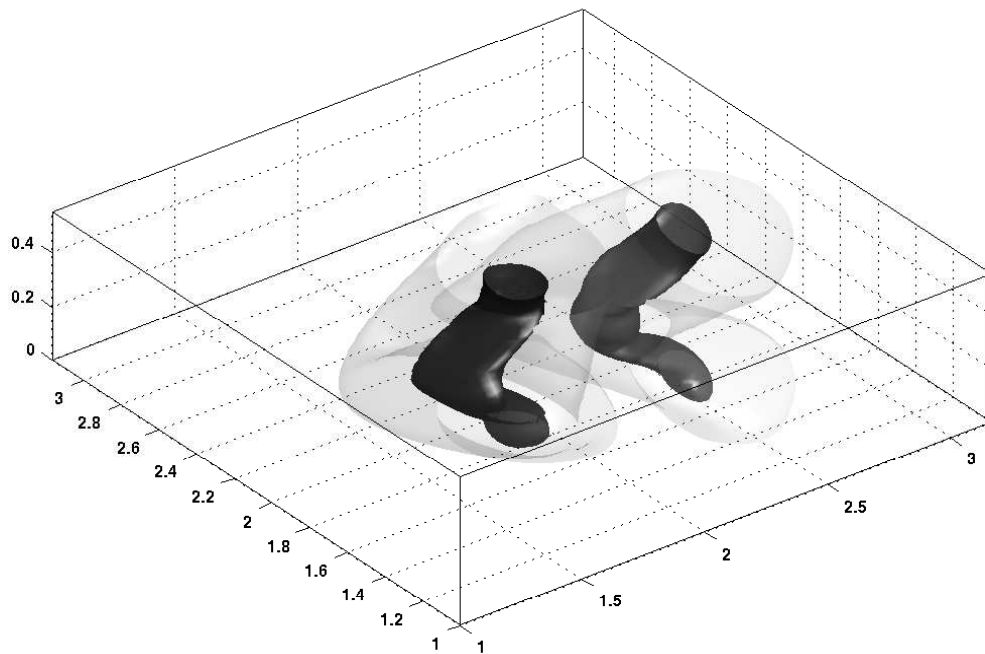


Figure 1: Nonlinear simulation of counter-rotating vortices bended by the zigzag instability $F_h = 1.33$, $Re = 2000$. This plot presents the first step ($t = 9$) when the instability grows exponentially without nonlinear saturation.

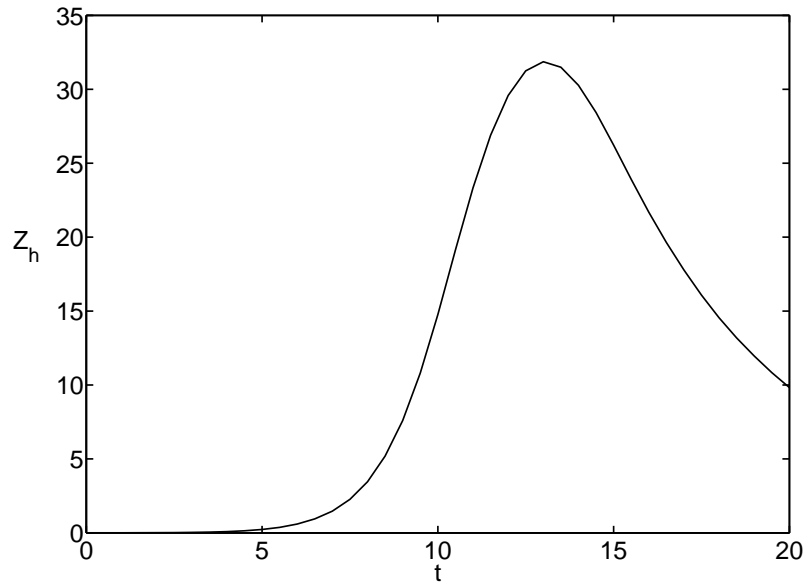


Figure 2: Evolution of the production of vertical shear followed through the creation of horizontal enstrophy Z_h . In a first step until $t = 13$, Z_h grows exponentially because of the development of the zigzag instability. Because of the production of intense vertical shear, this growth is stopped in a second step when vertical viscous effects become important.

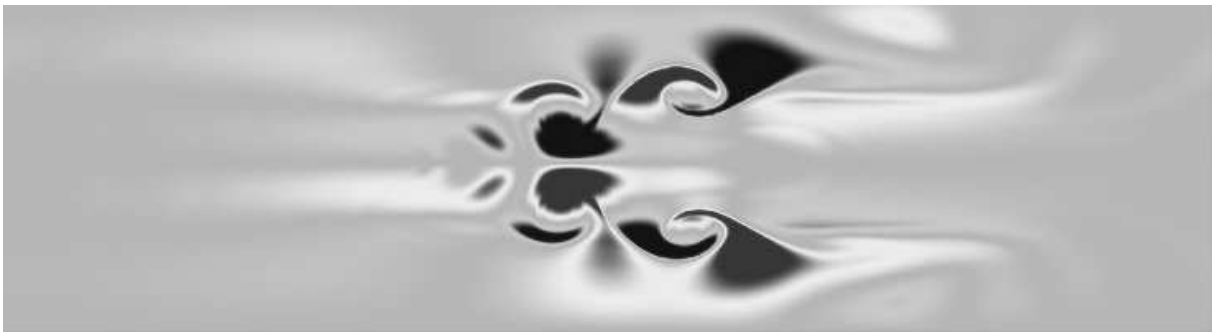


Figure 3: Vertical crosscut of the density perturbations clearly showing the apparition of Kelvin-Helmholtz instabilities ($t = 11$). This occurs for flows with sufficiently high initial values of $F_h^2 Re$. Here $F_h = 1.33$ and $Re = 6000$. In this situation, the vertical shear which is created by the zigzag instability is intense enough to lead to Kelvin-Helmholtz instabilities before being dissipated through viscous effects.