INTRINSIC MODES IN WEDGE SHAPED OCEANS

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Abstract — *We know that the Intrinsic Mode concept, recently introduced for the study of acoustic wave propagation in a penetrable wedge (Arnold and Felsen, J. Soc. Am., 76, 850–860,1984) can be efficiently computed using the FFT algorithm. In many circumstances, such as the prediction of field values along a single row of points parallel to the ocean surface, this method is much more efficient than conventional parabolic–equation and coupled–mode–theory methods, which generate mostly redundant data over entire transverse cross–sections. We present comparisions of acoustic field data generated both by Intrinsic Mode theory, and by the marching algorithms PEM and BPM, which indicate comparable accuracy, though the former method is much more efficient.*

1. INTRODUCTION

The simplest range dependent environment — the uniformly tapered waveguide — is of significant interest in both integrated optics and ocean acoustics. Conventional Adiabatic Mode theory cannot adequately explain the transition from the guided to the radiative regime (i.e. cut–off). However, extensions of Adiabatic Mode theory have been explored [1] which yield uniform asymptotic representations of the field as the observation point traverses the Adiabatic Mode cut–off region.

Recently, attention has been focused on spectrally synthesised fields, and in particular the source free solution of the homogeneous planar wedge environment depicted in Figure 1 (the Intrinsic Mode [2,3]). Self–consistent superposition of plane waves allows global spectral objects to be constructed which adapt to the slowly varying structure without any mutual coupling. Numerical evaluation of these Intrinsic Mode fields has been investigated by several authors [4,5]. However, these computations have required substantial computer time to generate a global field structure.

In this paper an efficient numerical scheme for calculating Intrinsic Mode fields, based on the Fast Fourier Transform (FFT) [6], is formulated, which is significantly faster than previous methods. Practical measurement of acoustical fields occurs along trajectories parallel to the ocean surface and theoretical approximate models (e.g. Parabolic Equation Method [7]) for this measurement generate large redundancy and are consequently very inefficient. As will be demonstrated the Intrinsic Mode formalism can generate exact field solutions in directions parallel to either interface without redundancy and are as a consequence significantly faster.

2. THE INTRINSIC MODE.

Synthesis of the Intrinsic Mode follows a well established philosophy [3]. It is the intention here to show its evaluation by Fast Fourier Transform techniques, and as such the finer points of its construction will not concern us here. It is sufficient to note that the Intrinsic Mode for the wedge shaped ocean in

Figure 1 has the form

$$W_q(\underline{x}) = \begin{cases} \displaystyle\sum_{-+} \int_C A^{\pm}(\theta) e^{-inkr\cos(\theta\pm\chi)} d\theta & \underline{x} \in X \\ \displaystyle\int_C A^{-}(\theta)\left[1 + e^{i\Phi_1(\theta)}\right] e^{-in_1 kr\cos(\theta_1-\chi)} d\theta, & \underline{x} \in X_1 \end{cases} \tag{1}$$

where θ_1 denotes the refracted angle at the lower boundary, defined by $n\cos\theta = n_1\cos\theta_1$ and

$$A^{\pm}(\theta) = \exp i\left[\frac{1}{2}\left[\Phi(\theta)\pm\Phi(\theta)\right] + \frac{1}{2\alpha}\int_{\theta_c}^{\theta} \Phi(s)ds - \frac{q\Pi\theta}{\alpha} + E(\theta) - \frac{1}{2}\Phi(\theta)\right] \tag{2a}$$

$$E(\theta) = \frac{1}{2\alpha}\sum_{\substack{-\infty \\ p\neq 0}}^{\infty} \int_{\theta_c}^{\theta} \Phi(s)\, e^{\frac{ip\Pi(s-\theta)}{\alpha}} ds \tag{2b}$$

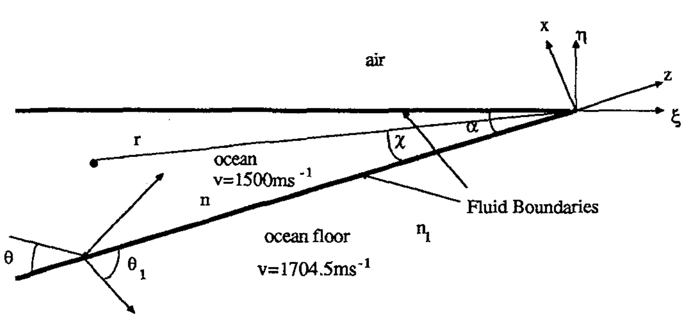
$$\Phi(\theta) = \Phi_u(\theta+\alpha) + \Phi_1(\theta)$$

where θ_c is the critical angle for the lower boundary ($\theta_c = \cos^{-1}(n_1/n)$). The term in square brackets in the integrand of (1) is the Rayleigh–Fresnel transmission coefficient at the lower boundary. The remainder term $E(\theta)$ is asymptotically small [2] and is neglected in this approach although its inclusion is a simple matter.

There are two types of Intrinsic Mode, each corresponding to a particular choice of the infinite contour C, since there are two inequivalent ways to construct contours satisfying the conditions of exponential decay at the contour end–points in the complex θ–plane. These two mode types are linearly independent; one of them is a purely outgoing wave as the observation point $\underline{x}$ moves out to infinity in the wedge sector, and the other is a 'second solution', required because the Helmholtz equation is second order in the space derivatives. These wave types are analogous to the $H_m^{(1)}$ and J_m – type Bessel functions which appear in the exact solution to the problem of a wedge with perfectly reflecting boundaries.

The application of FFT schemes requires a finite interval, (a,b) say, of the complex contour C. The major contribution to the field will occur from homogeneous plane–waves (real θ) and as such the finite range of integration will lie on the real θ axis. To be sure of obtaining all upslope propagating plane waves the interval along the real axis must be $(0,\Pi/2)$.

Figure 1: The Jensen -Kuperman Wedge Environment



3. THE FFT EVALUATION.

Functions of the form

$$F(x) = \int_0^T f(\beta)e^{i\beta x}d\beta \tag{3}$$

can be calculated efficiently by using standard FFT routines, which exploit the periodic nature ($e^{i\beta x}$) of the integrand. The output of a standard FFT routine is calculated using the butterfly method [6] to give,

$$F(k/T) = \frac{1}{\sqrt{N}}\sum_{j=0}^{N-1} f_j e^{\frac{i2\Pi jk}{N}} \tag{4}$$

where k is an integer in the interval (0,N) and N is the number of points in the discretised β- space which approximates the integral of (3) over the interval (0,T). The output spectrum is in the form of points in configuration x- space which are 1/T apart. To decrease the size of the interval in x- space it is possible to add zeros to the β- space domain to create a larger pseudo interval. Using this method a variable spacing in configuration space may be introduced without generation or loss of information.

Transferring to the Cartesian coordinate systems depicted in Figure 1 it is possible to represent the field globally by either of two coordinate frames ((x,z) and (η,ξ)). Indeed in principle it is possible to define the Intrinsic Mode in any rotated and translated coordinate frame of reference. However, consideration of the global nature of the field prompts the choice of the coordinate axes in Figure 1, allowing plane wave representations of the form,

$$e^{-inkr\cos(\theta\pm\chi)} = e^{ink(z\cos\theta\pm x\sin\theta)} \qquad \underline{x} \in X \tag{5a}$$

$$e^{-in_1kr\cos(\theta_1-\chi)} = e^{inkz\cos\theta - ink_1x\sin\theta_1} \qquad \underline{x} \in X_1 \tag{5b}$$

obtained by applying Snell's law at the lower boundary. As θ ranges over the interval (0,Π/2) $nkz\cos\theta$ remains real and applying the substitution $2\Pi s = nk\cos\theta$ the Intrinsic Mode becomes

$$W_q(\underline{x}) = \int_0^{\frac{nk}{2\Pi}} \frac{2\Pi K(\theta,x)}{nk\sin\theta} e^{i2\Pi zs} ds \qquad \underline{x} \in X \text{ or } X_1 \tag{6}$$

where $K(\theta,x)$ is obtained from (1) with the substitutions of (5) for observation points in the ocean and ocean floor. Efficiency can be improved by calculating the integrand of (6) independently of the coordinate axes. The consequence of this separation dramatically decreases run times, as the rate determining step in the calculation is the evaluation of $A^{\pm}(\theta)$.

RESULTS AND DISCUSSION

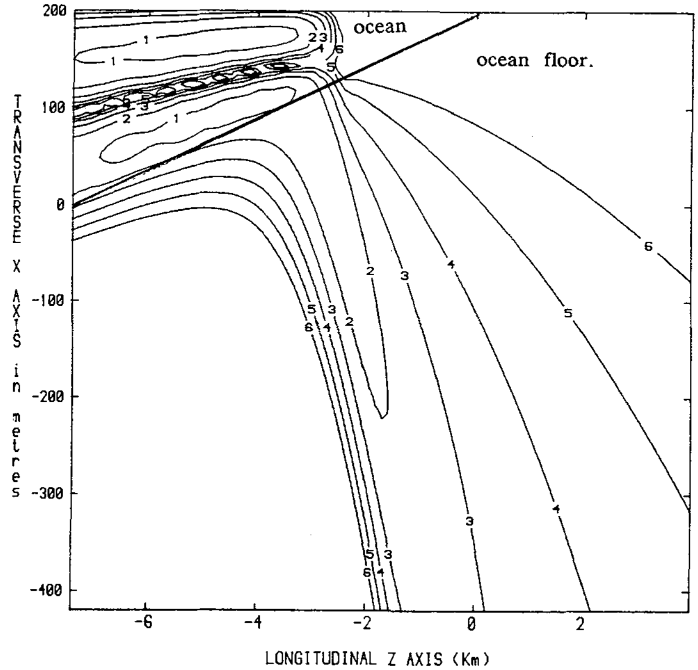
Our analysis shows that the Intrinsic Mode concept can be very efficiently implemented _via_ the FFT algorithm. The efficiency arises partly from the FFT itself, and partly from the fact that, at any given cross- section of the wedge sufficiently far from source regions, the total field can be represented as a superposition of a _finite_ number, N, distinct Intrinsic Modes, slightly greater than the number of _bound_ local normal modes supported at that cross- section. N is quite small ($\approx$3) in typical benchmark models of the wedge- shaped ocean [8].

While our analysis here has concentrated on the benchmark geometry of the wedge with plane boundaries (see Figure 2), waveguides guides with nonplanar boundaries can also support Intrinsic Modes, and in these cases also the Intrinsic Mode has the structure of a Fourier transform in the nominal waveguide plane. Similar advantages will therefore follow from FFT evaluation. Where the media forming the waveguide are piecewise homogeneous, the Intrinsic Mode accounts for local- normal mode coupling, and is uniform at cut- off.

We envisage the principal application of Intrinsic Modes to be as 'patching' transition functions in adiabatic- mode or coupled- mode models of acoustic wave propagation in shallow oceans of nonuniform depth, applied in order to progress an adiabatic- mode through cut- off without incurring the mathematical singularities which result from direct application of uncorrected nonuniform adiabatic- mode propagation. Secondly, the FFT algorithm makes our theory very advantageous when the acoustic field data is required to be predicted over a line (or plane) parallel to the ocean surface; this occurs, for example, when towing a hydrophone behind a ship in oceangraphic surveys. Alternative numerical methods, such as PEM, exhibit high degrees of redundancy in these circumstances since they must generate the field over an entire cross-section before advancing to the next, even though only one point per cross-section is actually required for the final computed data set.

Figure 2 The 2nd Intrinsic Mode in the Jensen-Kuperman Ocean.

$v = 1500 ms^{-1}$, $v_1 = 1704.5 ms^{-1}$, $\alpha = 1.55^{\circ}$



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