# A model of theory-practice relations in mathematics teacher education

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*The paper presents and discusses an ATD based (Chevallard, 2012) model of theory-practice relations in mathematics teacher education. The notions of didactic transposition and praxeology are combined and concretized in order to form a comprehensive model for analysing the theory-practice problematique. It is illustrated how the model can be used both as a descriptive tool to analyse interactions between and interviews with student teachers and teachers and as a normative tool to design and redesign learning environments in teacher education, in this case a lesson study context.*

**Keywords**: Anthropological theory of the didactic, teacher education, lesson study.

## THE THEORY-PRACTIC PROBLEMATIQUE

Establishing coherence between theory and practice is one of the main challenges in mathematics teacher education (e.g., Bergsten, Grevholm, & Favilli, 2009). In Denmark more than four out of ten student teachers experience a lack of coherence between the teaching of general educational science and didactics taking place at the university college and the practice of teaching in schools (Jensen et al., 2008). Throughout the last decades teacher education has become increasingly academic – which can be seen as positive – but concurrently, the practices at schools have become much more challenging due to increasing social and ethnic segregation, which affect schools particularly in disadvantaged neighborhoods. Therefore, many student teachers tend to focus on practical teaching tools rather than academic theories. This development causes a risk of a widening of the gap between theory and practice in teacher education.

The theory-practice divide can be regarded from (1) theory to practice or (2) from practice to theory. Ad (1) the questions are: How can theoretical knowledge be utilized to analyze and develop teaching practice in schools and how do we create a shared frame of reference from teaching practice to interpret the theory? Subject matter knowledge, pedagogical knowledge and pedagogical content knowledge are taught separately at university colleges but are in reality inextricably entwined with each other. The challenge is how to create interplay between the academic theories of mathematics and pedagogy and teaching practice in teacher education. It is crucial to create this interplay in order to legitimize the theoretical education and to place school knowledge in a wider context.

Ad (2) the teaching practice must be made visible and handled as the main object of discussion and theorization in the teacher education. This is necessary in order to ensure that student teachers' learning in and from teaching practice is connected to the theoretical education and brings about a critical view on the theories and research from a practical point of view.

These complex theory-practice relations in teacher education calls for a model, which can be used to describe and analyze the interplay between mathematical and didactical knowledge; teaching practice and learning in both teacher education and mathematics teaching in school. In particular, it is important that such a model can help differentiating between the different kinds of theory-practice relations in teacher education.

The aim of the research project behind this paper is to answer the following two research questions:

1) What different kinds of theory-practice problems appear in mathematics teacher education – according to the student teachers?

2) How can these theory-practice problems be conceptualized and analyzed within a model based on *The Anthropological Theory of the Didactic* (ATD)?

In this paper, the focus is on how the model can be used as a tool for analyzing empirical data from a lesson study project with teachers and student teachers. However, at first, the model evolved will be presented and discussed. The paper is round off with discussion of the benefit of the model in analyzing theory-practice relations in mathematics teacher education and on how such analyses can inform the design and use of lesson studies in teacher education.

## A MODEL OF MATHEMATICAL TEACHER EDUCATION

ATD (Chevallard, 2012) provides an epistemological framework for mathematical knowledge. In ATD mathematical knowledge, regarded as a human activity including teaching and learning mathematics, is modelled by mathematical and didactical *praxeologies* (Winsløw & Madsen, 2008). Praxeologies consist of a practice block (*praxis*) regarding the questions *what to do* and *how to do it* (know-how) and a theory block (*logos*) regarding *why to do it* (know why). In addition to this, ATD models the *didactical transposition* of mathematical knowledge from *scholar mathematics* mainly evolving at universities to *knowledge actually taught* and *learnt* in schools (Bosch & Gascón, 2006, p. 56). The didactical transposition is divided into two steps. The first step is the *external didactical transposition* from *scholar mathematics* to *knowledge meant to be taught* – the mathematical knowledge as it is described, for example, in curriculum and textbooks. This step is often performed by people outside the school. The second step is the *internal didactical transposition* from *knowledge meant to be taught* to *knowledge actually taught* – this step is every day work for teachers.

The two concepts, praxeology and didactic transposition, both bring central theory-practice relations into focus – the first one inside an institutional frame (e.g., the school) and the second in a broader context between institutions. Together they provide a comprehensive picture of teacher education in mathematics, which can be usable to point out and analyse problems and constraints as the theory-practice problematique.

In the model below the two concepts are combined to form a model for analyzing the theory-practice problematique in teacher education (Figure 1). In my research the model is intended to be a tool for both descriptive and normative analyses. At first, the model is used descriptively to analyze different kinds of empirical data from two lesson study projects in connection with teacher education. On this basis, the model will be used normative to propose new ways to organize teaching practice, preparatory education and the theoretical education at the university college to improve the coherence between theory and practice in teacher education.

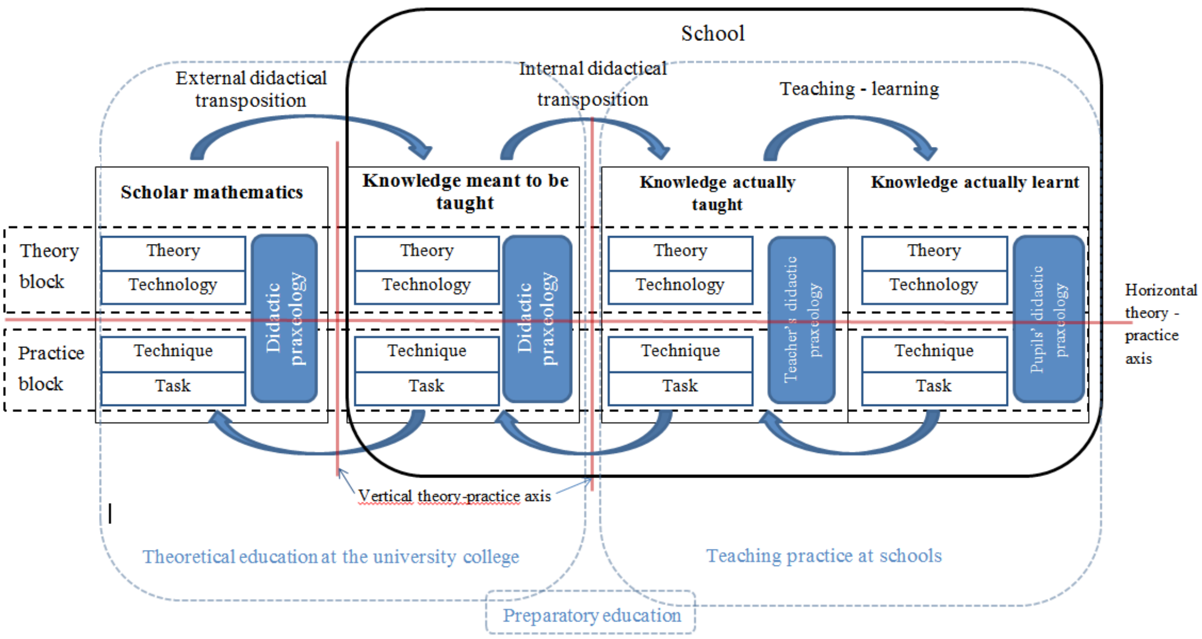


**Figure 1:** Teacher education model

The model consists of four columns containing the four kinds of knowledge in the didactic transposition. Each kind of knowledge is described by a mathematical praxeology with theory, technology, technique and task (see Winsløw & Madsen, 2008, for further details) depict with white boxes and a didactic praxeology, also with a theory and practice block, depict as blue boxes in Figure 1. By collocating the model and teacher education practice three different, pivotal theory-practice problems can be located – occurring in different forms. These are emphasized by red axes – two vertical and one horizontal axis.

The horizontal axis is dividing the practice blocks and the theory blocks. This axis stresses the divide between practical, procedural mathematics with emphasis on techniques to carry out tasks and theoretically *doing mathematics* by combining techniques and concepts, arguing, proving etc. The transcendence of this barrier is a crucial point for mathematical education – the higher level of abstraction in the theoretical block is a necessity but also a very difficult barrier to almost all pupils. Consequently, this axis is a significant problem area for teacher education both with regard to student teachers learning scholar mathematics and pupils learning mathematics at school and the relation between practice and theory block is an appropriate model in both cases.

The two vertical theory-practice axes are dividing, respectively, the scholar mathematics and knowledge meant to be taught and knowledge meant to be taught and knowledge actually taught. The divide in the first axis is treated at the university college. Comparison of scholar mathematics and knowledge meant to be taught is again highly relevant in teacher education to analyze what and why specific content is or is not selected for curriculum. It is pivotal for student teachers to be critical to this selection and to question the decisions in curriculum or textbooks. The arrows at the base of the model pointing "back", for example, from knowledge meant to be taught to scholar mathematics stresses that knowledge meant to be taught or actually learned can be taken as a starting point for analyzing the mathematical knowledge on the previous levels in the system. The latter of the vertical axes is dividing the theoretical education taking place at the university college and teaching practice at schools. To combine these two, university colleges often organize preparatory education as a special forum, depict as a small box in the bottom of the model.

The *internal didactical transposition* from knowledge meant to be taught to knowledge actually taught is everyday work for teachers and thus obvious content in mathematical teacher education.

The two columns to the right are a little different compared to the other kinds of knowledge. The relation between knowledge actually taught and knowledge actually learnt cannot offhand be described as a theory-practice problem because both are a part of the teaching practice at schools – the knowledge *actually* taught and learnt. Of course, teaching and learning can be described and analyzed by theoretical tools but the interplay at schools is a practice matter. As the transposition takes place inside school it is a part of the internal transposition but knowledge actually taught and knowledge actually learnt are closer connected and appears in a more direct interrelationship than the other kinds of knowledge. Student teachers are supposed to react to pupils' communication and learning, for example, during a dialogue in the classroom and adapt the teaching to the individual pupil or the specific class. Knowledge actually taught and knowledge actually learnt can be theoretically analyzed separately but are intertwined in practice. Therefore, the two kinds of knowledge are not separated in the model, but have a common borderline regarded as the interplay between the pupil's knowledge and the knowledge presented by the teacher in the form of the teaching environment presented.

## THE LESSON STUDY PROJECT

The next section is an analysis of a group of two teachers and three student teachers' learning outcome from a lesson study project on the basis of the ATD-model. The lesson study was conducted in autumn 2013 in two classes grade 6 and 7 and the title was *"Similar – what does it mean?"* It was a part of a bigger lesson study project with the title *Trigonometry and inquiry based learning* involving 29 student teachers and 17 teachers conducted by a colleague and me. The empirical data from the lesson study consists of a lesson plan, video recordings of the two completions of the lesson, two 45 minutes interviews with one of the teachers and one of the student teachers and an article written by the student teachers. After the lesson study project I formulated an interview guide and accomplished the following data analysis on the basis of the ATD-model with a special focus on the three theory-practice axes.

### Lesson study

Lesson Study is a Japanese form of action based development of teaching and teachers' and student teachers' teacher knowledge. In Japan, Lesson Study is an integrated part of both teacher education and teaching development in elementary school (Stiegler & Hiebert, 1999).

The ingenious and yet simple idea of Lesson Study is that the participants – student teachers and/or teachers – consider substantial didactical questions through mutual preparation, completion, analysis and reflections on one single lesson. Participants' observations and subsequent reflections on the pupils' learning and from this the didactical theme are crucial elements in the format. In the concrete project the student teachers studied trigonometry and inquiry based education at the University College before the project to be well-prepared to cooperate with the teachers. Together with the teachers, they prepared the lesson and formulate focus points for the observations to ensure that the observations are targeted at the didactical theme. A central element in Lesson study is the written *Lesson Plan* encompassing i.a. deliberations on mathematical, didactical and pedagogical aims of the lesson and hypothesis on pupils' strategies to solve the problems they are faced with. After a minimum of four hours of preparation one of the student teachers taught the lesson while the rest of the participants observed on the basis of the focus points. The lesson was evaluated immediately after the completion on the basis of the observers and the teacher's observations after a carefully worked-out plan. The evaluation resulted in suggestions to change the lesson and improvement of the lesson plan. Afterwards, the lesson was taught by a new student teacher in a new class immediately followed by an evaluation meeting where the second completion and the entire lesson study process was evaluated. The lesson plan was edited and the gained experience was described and discussed. At last, the student teachers wrote an article for *Matematik* – the Danish journal of mathematics teacher. It must be emphasized that the learning outcome from the lesson study not only – and not even mainly – relates to the lesson in question. On the contrary, lesson study is suitable to work with pedagogical and didactical problems on a generally level. The concrete and empirical basis opens up new possibilities to confront didactical and pedagogical principles with teaching practice at schools (for further details see, e.g., Lewis, 2002).

### Data analysis: The lesson plan

The lesson plan is divided into three sections: First, some practical information concerning the participants, who taught the lessons, the name of the school, dates for completion of the lesson and the classes involved. The second part encompasses the tittle and aims of the lesson, competencies involved and working method. The last section is a detailed plan of the lesson containing mathematical focus point and learning goals of the lesson, a timetable, key question, teaching resources and useful tips for the teacher.

The lesson starts with a 10 minutes introduction to *geometric similarity* on the basis of every day examples of similar and not similar objects like a golf ball and a football, different sizes of Toblerone packaging (chocolate), enlarging/reducing in a photocopier and a pony and an Arab horse (not similar). After the introduction, the pupils receive a right-angled triangle cut of cardboard and the teacher asks the key question: *"You shall pretend that you are a photocopier and draw an enlarged and a reduced copy of the triangle"*. This is the main mathematical task *t* of the *knowledge actually taught*. When the pupils have drawn the two triangles they must contact the teacher. The teacher then asks them two questions: *"How did you construct the triangles?"* and *"How can you convince me that the two triangles are similar to the one cut of cardboard?"* The two questions encompass the transcending of the horizontal theory-practice axis from the practice block to the theory block in knowledge meant to be taught. The teacher's didactic praxeology in connection to this mathematical praxeology is therefore a key issue of the lesson.

The crucial mathematical praxeology to be developed in the lesson study is based on the type of task T: *Given a right-angled triangle, how can you reduce/enlarge the size without changing the form?* A possible, predictable – and desirable – technique $\tau$ is to copy two or three angles from the cardboard triangle for instance by putting it on top of the paper and draw the angles and then reduce/enlarge the length of the sides. The technology $\theta$ to be realized by the pupils is firstly, that equiangular triangles are similar and secondary; the ratios between the lengths of equivalent sides are constant. Theory $\Theta$ – in this case the mathematical definition of similarity – is framing and justifying technology.

The lesson plan also points out some pivotal didactic praxeologies. One substantial didactical praxeology is concerning the inquiry based education mentioned in the category *mathematical working methods: "working in pairs – inquiry based education*" (IBE). IBE is concerned with the teaching-learning relation in the model – a theoretical idea about how pupils learn and from this how to teach. The example shows the delicate interplay between the pupil's and the teacher's didactic praxeology. The appertaining type of task in the teacher's didactic praxeology is how to set up a learning environment that makes the pupils investigate the mathematical task. The task is not explicitly mentioned in the lesson plan but two different techniques to solve the task appear in the following quotes: *"The pupils work inquiring with concrete materials and get the opportunity to reason on their own"* and *"Tips for the teacher: Be careful not to unveil the points"*. So, the two main didactical techniques are to use concrete materials and to give the pupils opportunity to work out their own solutions (in pairs) without a standard procedure presented by the teacher.

The example shows how the model captures underlying mathematical and didactical considerations and the relations between these. In this case, the model is primarily used descriptively to analyse the lesson plan but it can as well be used normatively for instance to improve the design of the lesson plan template in the example about the ratios between the lengths of the sides by stressing the connections between mathematical and didactical praxeologies or type of task, technique and technology.

**Video recordings of the lessons**

The video recordings show that the student teachers to a great extent conduct the lesson as it is described in the lesson plan. They have experience with lesson study and know that this is important to focus the attention on the teaching instead of the teacher. During the section of the lesson where the pupils work with the problem in pairs they stick to the manuscript of the lesson, for example, *"Be careful not to unveil the points"*, and pose the planned question. For instance, in the following situation in grade 6:

| | |
|---|---|
| Pupil 1: | This one is double size |
| ST: | How can you convince me, that it is the same triangle? Can you argue that they are similar? |
| Pupil 1: | It has the same shape – and it has three sides |
| Pupil 2: | And it is right-angled |
| ST: | Yes. But so is this triangle (the teacher shows a triangle from another group). And your triangles are not similar to this one? |
| Pupil 1: | No |
| ST: | No, but they both have a right angle and three sides. Try to find out what the similar triangles have in common but these have not. Think about it... (The student teacher leaves) |

The student teacher's first question is almost exactly quotation from the lesson plan. This question is difficult to answer to the pupils. Nevertheless, Pupil 1 refers to "same shape" as a colloquialism but unfortunately, the teacher do not respond to the suggestion and so the pupil do not get the opportunity to create a link to the mathematical concept – equal angles. This is the task of the didactic praxeology – to extend their understanding of the everyday word similar to a more exact mathematical interpretation. The example (and others alike) shows that the question does not encourage the pupils to investigate mathematical properties about the similar triangles and thereby get an opportunity to become acquainted with the theory block of the mathematical praxeology. The technique to solve the didactical task seems to fail. Maybe as a consequence of this, the student teacher improvises and reformulates the question: *"Try to find out what the similar triangles have in common but these have not."* This question is not mentioned in the lesson plan but it leads the pupils to examine mathematical properties because the question is posed in mathematics. An obvious answer to the question is that similar triangles have angles in common but ratios of the length of sides are not in the same way immediate obvious for pupils at this age. A new didactical task is therefore how the teacher can pose questions to lead the pupils to examine the ratios of the length of sides without "unveiling the point"? Analyzing the situation by means of the model could for example lead to a question like *"What will happen if you multiply the length of the three sides with the same number – 2 for example?"* The example shows that a problem concerning the didactic praxeology requires an analysis in details of the appertaining mathematical praxeology.

The video recordings show that the student teachers are very determined to follow the lesson plan as it is planned by the participants. The comprehensive preparation of the lesson and the very close connection to the theoretical education gives the student teachers an opportunity to try out their theoretical knowledge – both didactical and mathematical – in practice. Because they stick very carefully to the lesson plan there is a close connection between knowledge meant to be taught and knowledge actually taught and between the mathematical and the didactical praxeology – this is a crucial challenge in teacher education. Obviously, this challenge should be taken up in teaching practice but student teachers often find this very isolated from the theoretical education at the university college. In teaching practice the student teachers are "forced to act" – they have to teach a fixed number of lessons each week. Therefore, they often experience teaching practice as complex and stressful and fall into short-lived performance without coherence to their learning outcome from the theoretical education.

**Interviews**

Dialogue and working relationship between teachers and student teachers are – off course – important learning resources about school practice for student teachers. The interviews show that both teachers and student teachers experience significant differences between the dialogue involving teachers and student teachers in the lesson study compared to the usual teaching practice situation:

> Teacher (about teaching practice): Usual, when you have student teachers, it is vulnerable. Very often, you tell them what they did wrong or what they shall be aware of next time in the class instead of sticking to the point, the lesson, the content. (...)
>
> Teacher (about lesson study): Focus is on the lesson and not on the student teachers. We are not supposed to supervise them. We discuss what is working and what is not working about the lesson. We share a common responsibility to make the lesson work. We don't evaluate the student teachers but the lesson.
>
> Student teacher: In teaching practice, the teacher watches you when you teach, whereas in lesson study we are equal. We should all participate in the preparation of the lesson and we could all contribute to the lesson.

In teaching practice the student teacher usually prepare the teaching and teach a single lesson, while 1–3 of his or her fellow students and the teacher observe the lesson. Afterwards, the teacher supervises the student teacher in very close connection to the student teacher's presentations and interactions with the pupils in the lesson. The student teacher then tries to *"correct the mistakes"* before the next performance – some descriptions by student teachers and teachers indicates an inappropriate "trial and error" method. The strong focus on the student teacher's performance emphasizes the practice block of the teacher's didactic praxeology and – to a lesser extend – the teaching-learning relation in the ATD-model. According to both teachers and student teachers, it is unusual to discuss didactical and mathematical theory, curriculum and other topics in connection with the teaching on a general level – the two columns to the left in the model are almost absent in the dialogue. The interviews show that the teacher's didactical praxeology in knowledge actually taught is most often disconnected from both the appertaining mathematical praxeology and the mathematical and didactical praxeologies in the other columns. This is evident in many of the dialogs between teachers and student teachers in connection with teaching practice. Such practice off course implies a risk of widening the gap between student teacher's experience of theory and practice during their teacher education.

The interviews show two main differences between the dialogue between teachers and student teachers in connection with usual teaching practice and lesson study. Firstly, the dialogue in lesson study take place both before and after the teaching and especially the very long time spent preparing the lesson was emphasized as fruitful. The common preparing together with the lesson plan template makes the participants discuss and consider knowledge meant to be taught, the internal didactical transposition and the interplay between mathematical and didactical praxeologies. Secondly – as stressed by the teacher in the quote above – focus is on *teaching* and not the *teacher* in lesson study. This implicates for instance that knowledge actually learnt to a much higher degree is included in the dialogue in connection with lesson study than it is in the dialogue in connection with teaching practice and thus, the interplay between knowledge actually learnt and knowledge actually taught can be examined, discussed and related to knowledge meant to be taught.

There is a very clear consensus between teachers and student teachers that the dialogue in connection with lesson study to a much higher degree than the dialogue in connection with teaching practice includes a broader range of pivotal problems in teaching and learning mathematics. As a consequence, theory-practice axis in the model are treated and transcended more often.

## CONCLUSION

The ATD model points out three different theory-practice problems in mathematics teacher education. It is crucial to put focus on all three axes and give student teachers opportunities to establish coherence between theory and practice in connection to the three axes.

Through different examples from a lesson study it is shown, that the model can be a fruitful tool to analyse teaching and learning contexts. Firstly, the model can be used as a descriptive tool to analyse and criticize planned teaching (lesson plan), actually completed teaching and the participants' experiences of the teaching with a special focus on theory-practice problems. Secondly, the model can be used as a normative, prescriptive tool for making adjustments to or changes in didactical designs for teacher education.

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