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Universal Log-Periodic Correction to Renormalization Group Scaling for Rupture Stress Prediction From Acoustic Emissions

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Abstract. — Based on the idea that the rupture of heterogeneous systems is similar to a critical point, we show how to predict the failure stress with good reliability and precision ($\approx 5\%$) from acoustic emission measurements at constant stress rate up to a maximum load 15-20% below the failure stress. The basis of our approach is to fit the experimental signals to a mathematical expression deduced from a new scaling theory for rupture in terms of complex fractal exponents. The method is tested successfully on an industrial application, namely high pressure spherical tanks made of various fiber-matrix composites. As a by-product, our results constitute the first observation in a natural context of the universal periodic corrections to scaling in the renormalization-group framework. Our method could be applied usefully to other similar predicting problems in the natural sciences (earthquakes, volcanic eruptions, etc.)

Acoustic emissions (AE) are mechanical waves produced by sudden movement in stressed systems, occurring in a wide range of materials, structures and processes, from the largest scale (seismic events) to the smallest one (motion of dislocations). It is a rather delicate technique to use and interpret since each loading is unique, tests the whole structure and can easily be contaminated by noise. Notwithstanding the development of numerous AE structural testing procedures [1,2], hopes to use AE as a reliable practical method for failure prediction have not been borne out.

AE differs from most other nondestructive testing (NDT) methods in that the AE signal has its origin in the material itself, not in an external source, and in that it detects movement, while most methods detect existing geometrical discontinuities. Thus, the general basis for developing a prediction scheme is to first determine which detected AE are relevant precursory phenomena (among the various causes such as crack nucleation and growth, fiber-matrix delamination, fiber rupture...) and then develop an algorithm which allows one to relate these precursors to the final rupture.

Here, we address the general case where global rupture is not controlled by a single event (nucleation and growth of a single crack), as occurs in a homogeneous system with very few defects, but rather by a succession of events, in general corresponding to diffuse damage, possibly culminating in the catastrophic growth of a dominating crack. In this case, we claim that failure can only be predicted by viewing rupture as a cooperative phenomenon: it is not through the separated analysis of the succession of the recorded hits that rupture can be predicted but by somehow synthetizing all the information embedded in these events in a global framework.

From a formal point of view, our approach to rupture prediction consists in viewing the final stage of rupture as a kind of critical point, which can be described by a fixed point in a renormalization group scheme (see below). This description is inspired from a wealth of recent work in the statistical physics community, mainly based on exact solutions of solvable models and on extensive numerical simulations, which have shown the existence of certain scaling laws up to a time of total rupture [3–6]. Our method is thus at the opposite of the view often advocated in the mechanical literature, which consists in modelling an (evolving) heterogeneous system as an effective medium characterized by average properties, including an increasing global damage parameter. These approaches are suitable only far from rupture and are unable to capture the specific N-body nature of the rupture critical point, and are therefore useless as predictors.

In order to fix ideas, we shall consider an industrial application on which our method has been tested extensively. The complexity of such systems can be taken as a test of the robustness of our technique. The systems are spherical tanks made of carbon fibers pre-impregnated in a resin matrix wrapped up around a metallic liner. We have tested different materials (carbon or kevlar fibers, different metallic (titanium or steel) compounds for the liner), as well as different tank dimensions (radius of 0.2 to 0.42 m). AE signals are obtained from three to six acoustic transducers (resonant frequency of 150 kHz) placed at equal distances on the equator of a given spherical tank by increasing at a constant rate (3 to 6 bar/s) the internal water pressure and thus the stress exerted on the tank. Figure 1 represents a typical data set of the instantaneous AE energy rate $\mathrm{d}E/\mathrm{d}t$ as a function of the applied internal pressure p up to the rupture threshold p_r, obtained by simple addition at each time step of the intensities measured on all operating transducers. Note the complex intermittent structure of the data, with quiet periods separated by bursts of widely different amplitudes, and the large increase of the AE energy rate on the approach to failure (occurring in the present case at $p_\mathrm{r} = 713$ bars).

In order to test the critical point concept on this particular set of systems, we checked for the existence of a power law,

$$\mathrm{d}E/\mathrm{d}t = E_0(p_\mathrm{r} - p)^{-\alpha} \tag{1}$$

where α is a so-called critical exponent. In all cases explored, we found that for p sufficiently close to p_r to within about less than 5%, the dramatic increase of AE energy rate on the approach to failure is indeed fitted very well by a power law (1), with an exponent $\alpha = 1.5 \pm 0.2$ independent of the specific sample or previous history as long as the critical zone (approximately in the interval $[0.95p_\mathrm{r};\ p_\mathrm{r}]$ has not been reached in preceding loads. The power law (1) was checked either by representing $\mathrm{Log}(\mathrm{d}E/\mathrm{d}t)$ as a function of $\mathrm{Log}(p_\mathrm{r} - p)$, using the measured p_r. This is illustrated in Figure 2 for three different tanks (and two different partitions for one tank) which exhibits clearly the asymptotic linear dependence when sufficiently close to p_r. We find that all data tend to a straight behavior whose slope determines α. We also studied $(\mathrm{d}E/\mathrm{d}t)^{-1/\alpha}$ (with $\alpha = 1.5$) as a function of p. Again, an asymptotic straight line is obtained whose intersect with the abscisse gave a very good estimate for p_r better than 1%. These two fitting procedures are not independent but put different weights on the data point. Therefore,

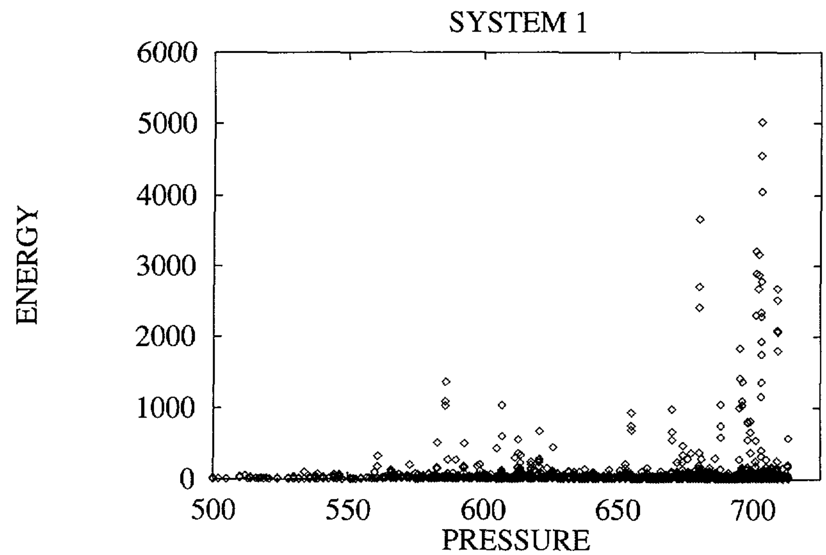

Fig. 1. — *Instantaneous AE energy rate* $\mathrm{d}E/\mathrm{d}t$ *in system 1 as a function of the applied internal* pressure p at constant pressure rate of 6 bar/sec up to the rupture threshold $p_{\mathrm{r}} = 713$ bars.

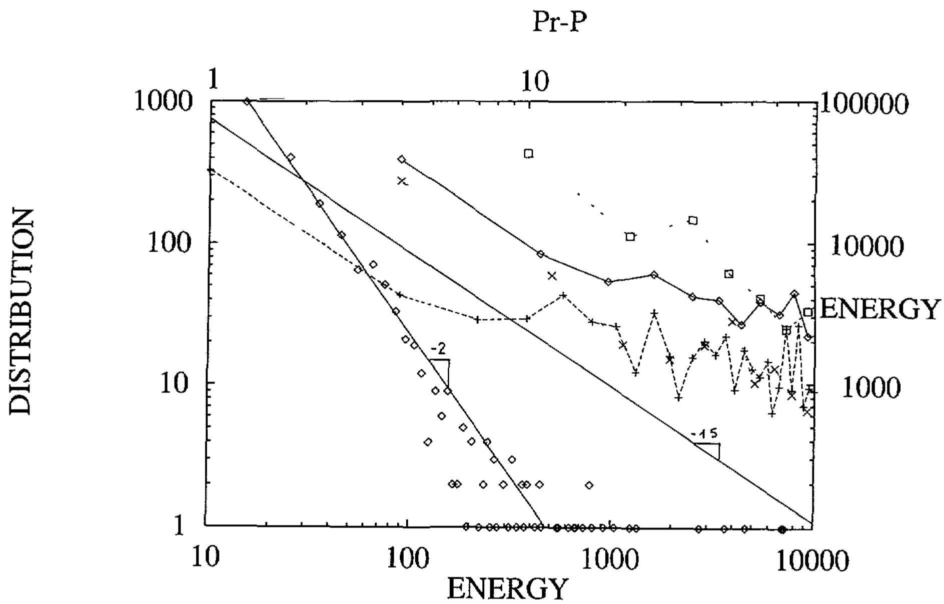

Fig. 2. — Upper right: $\mathrm{d}E/\mathrm{d}t$ in log-scale as a function of $p_{\mathrm{r}} - p$ in log-scale, using the measured p_{r} for each system, in three different systems: system 1 (□); system 2 (◇) and (+); system 3 (×), system 2 being represented twice for two different coarse-graining (◇): 11 intervals; (+): 27 intervals. The straight line has a slope $\alpha = 1.5$. Lower left: differential distribution $N(E_{\mathrm{b}})$ of burst energy E_{b} in log-log scale. The straight line has a slope of $\alpha = -2$.

verifying their consistency is important for checking the existence of the power law (1). As expected, the exponent α is found "universal"and equal to 1.5 and insensitive to the specific tank realization. It could however depend upon the tank components which control the nature of damage. Figure 2 also shows the differential distribution of burst energy 'E_b' which is also a power law $N(E_b) \sim E_b^{-(1+B)}$, with an exponent B consistently equal to 1 ± 0.2 for all the systems investigated. This law, which is reminiscent of the Gutenberg-Richter distribution of earthquake sizes [7], has been studied in detail in [8] in the AE context.

Armed with these empirical tests of the concept of critical rupture, prediction should in principle be possible by extrapolation of AE data using expression (1). Note that this scheme is similar to that proposed by Voight a few years ago to describe and predict rate-dependent material failure, based on the use of an empirical power law relation obeyed by a measurable quantity [9, 10]. However, this procedure is unpractical due to the narrowness of the domain of validity of the law (1) (critical region $[0.95p_r;\ p_r]$ in most explored cases), preventing a prediction at pressures less than, say, 0.97 p_r.

A useful scheme should allow one to make predictions at much lower pressures. Consider for instance an attempt to predict p_r by monitoring the AE up to, say, 0.85 p_r, i.e. up to about 610 bars in the case shown in Figure 1. Inspection of Figure 1 shows that, apparently, very little of the whole AE data is contained in the AE set up to 610 bars. Furthermore, the dependence of the rate of AE energy release as a function of the instantaneous applied pressure up to 610 bars has apparently nothing to do with a power law such as (1) and the prediction thus seems hopeless.

Our fundamental idea is that the concept of rupture criticality embodies more usable information than just the power law (1) valid in the critical domain. We argue that specific precursors outside the critical region can lead to "universal" recognizable signatures in the AE data of the final rupture. In order to identify these signatures, we first note that an expression like (1) can be obtained from the solution of a suitable renormalization group (RG) [11]. The RG formalism, introduced in field theory and in critical phase transitions, amounts to view the N-body problem as a succession of 1-body problems with effective properties varying with the scale of observation. It is based on the existence of a scale invariance of the underlying physics which allows one to define a mapping between physical scale and distance from the critical point in the control parameter axis: in our problem, the damage rate and therefore AE rate at a given pressure p are related to those at another pressure p' through a suitable non-linear transformation $p' = p_r - \phi(x)$ with $x = p_r - p$. The RG formalism then provides the general structure of the functional equation that the physical observable must obey (for the simplest case of a one-parameter RG):

$$\mu F(x) = F(\phi(x)) \tag{2}$$

For the sake of simplicity, we have introduced the notation $F(x) = (\mathrm{d}E/\mathrm{d}t)^{-1}$, the inverse of the AE energy rate, which goes to zero at the critical rupture point $x_r = 0; \mu$ is a real positive number. We assume that the function $F(x)$ is continuous and that $\phi(x)$ is differentiable. The connection between this formalism and the critical point problem stems from the fact that the critical point is on the attractor of the fixed point of the RG flow. Then, the function $\phi(x)$, which generates the so-called RG flow, is usually used to extract the qualitative behavior as well as the stability of fixed points and to deduce the corresponding critical exponents. Let us assume for simplicity that the critical point is not only on the attractor of a fixed point but is indistinguishable from it, as can be done by a suitable change of variable. Then, if $x = 0$ denotes a fixed point ($\phi(0) = 0$) and $\phi(x) = \lambda x + ..$ is the corresponding linearized transformation, then the solution of (2) close to $x = 0$ is given by equation (1) with $\alpha = \mathrm{Log}\mu/\mathrm{Log}\lambda$.

To go beyond this local analysis in the critical region, we are interested in the general solutions of equation (2). To get them, let us assume that $F_0(x)(=x^\alpha)$ is a special solution, then the general solution $F(x)$ is related to $F_0(x)$ in terms of a periodic function $p(x)$, with a period Logμ, as:

$$F(x) = F_0(x)p(\log F_0(x)) \tag{3}$$

Since $\log F_0(x) = \alpha \text{Log} x$, this leads to a periodic (in Log$x$) correction to the dominating scaling (1). Equivalently, this log-periodicity can be represented mathematically by a complex critical exponent, since $\text{Re}\left\{x^{\alpha'+i\alpha''}\right\} = x^{\alpha'}\cos(\alpha''\text{Log}x)$ gives the first term in a Fourier series expansion of (3). This expression thus introduces universal oscillations decorating the asymptotic powerlaw (1). The mathematical existence of such corrections has been identified quite early [12] in RG solutions for the statistical mechanics of critical phase transitions, but has been rejected for translationally invariant systems, since a period (even in a logarithmic scale) implies the existence of one or several characteristic scale which is forbidden in these systems. For the rupture of quenched heterogeneous systems, the translational invariance does not hold due to the presence of static inhomogeneities [13] and the fact that new damage occurs at specific positions which are not averaged out by thermal fluctuations. Hence, such log-periodic corrections are allowed and should be looked for in order to embody the physics of damage in the non-critical region.

It is interesting to mention the few known cases where such a behavior has been observed. Probably the first theoretical suggestion of the relevance to physics of log-periodic corrections to scaling has been put forward by Novikov to describe the influence of intermittency in turbulent flows [14]. Loosely speaking, if an unstable eddy in turbulent flow typically breaks up into two or three smaller eddies, but not into 10 or 20 eddies, then one can suspect the existence of a preferable scale factor, hence the log-periodic oscillations. A clear-cut experimental verification of the generation of log-periodic oscillations by discrete scale invariant fractals have been carried on on man-made Sierpinsky networks of normal-metal links, in which the normal-superconductive transition temperature presents log-periodic oscillations as a function of the applied magnetic field [15]. Complex fractal exponents have also been argued to describe the branching architecture of the mammalian lung [16]. To our knowledge, there is no experimental verification of this fact but renormalization group calculations of critical behavior of random dipolar Ising systems [17] and spin glasses [18] have shown the existence of complex critical exponents. These results taken very cautiously by their authors could be the signature of a spontaneous generation of discrete scale invariance due to the interplay of the physics (interaction) and the quenched heterogeneity. Boolean delay equations involving two time lags with an irrational ratio, used recently to model the climate variability, also exhibit superdiffuse behavior with log-periodic oscillations [19]. Here again, we have an example of a discrete scale invariance which is spontaneously generated, in the present case by the threshold dynamics and nonlinear feedback involved in the Boolean delay equations. Finally, it has been pointed out that vibration and wave properties on discrete fractal structures should be characterized by log-periodic corrections to the leading singular behavior for the density of states close to the band edges [20]. However, our results on these periodic corrections, that are presented below, are the first ones obtained in structures containing an uncontrolled heterogeneity and, to our knowledge, have not been observed previously in the context of rupture.

Figure 3 shows a fit (continous lines) obtained using the solution of equation (3) applied to two AE data sets (represented by the symbols) obtained on two differents tanks (systems 1 and 2). Here, as in Figure 2, we group the AE hits in time intervals of varying width in order to reduce the noise. To express the solution of equation (3), $p(x)$ has been expanded in Fourier series and we have kept only the dominant term:

$$\mathrm{d}E/\mathrm{d}t = E_0(p_\mathrm{r}-p)^{-\alpha}[1 + C\cos\{\phi + 2\pi \text{Log}(p_\mathrm{r}-p)/\text{Log}\lambda\}]. \tag{4}$$

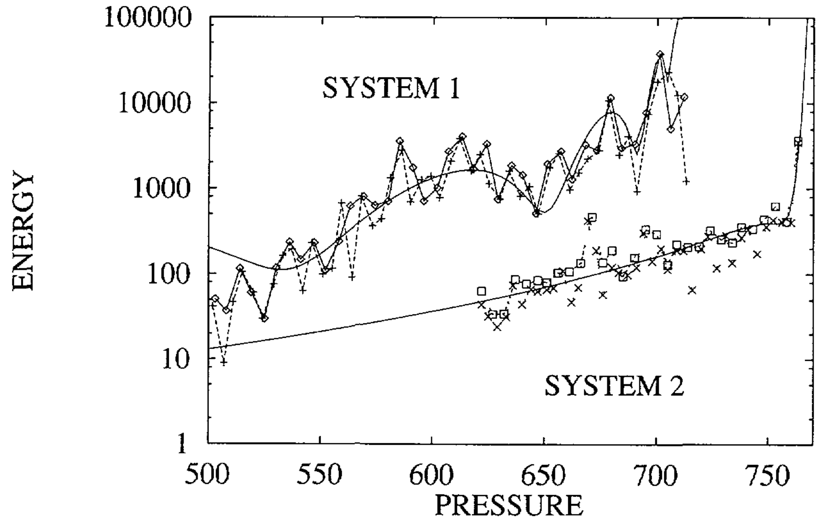

Fig. 3. — Theoretical fit using equation (3) shown in continous lines of two AE data sets (represented by the symbols) obtained on two differents tanks (systems 1 and 2). For each system, the different symbols corresponds to different coarse-graining.

The resulting mathematical expression has *a priori* six unknown parameters: a normalizing factor E_0, the critical exponent α, the pressure p_r at rupture, the amplitude C of the oscillatory correction, its period $\mathrm{Log}\lambda$ and phase ϕ. To have better conditioning, we impose $\alpha = 1.5$ obtained from our previous fit shown in Figure 2, since we expect it to be "universal" within a class of materials. This leaves us with five unknown variables to determine from a non-linear fit. We used the Levenberg-Marquardt method [21], which combines the steepest descent method with the inverse Hessian method. One can observe in Figure 3 the importance of the oscillatory corrections to the leading power law behavior for correctly accounting for the intermittent burst structure of the AE data shown in Figure 1. It is important to remark that the periodicity in $\mathrm{Log}(p_r - p)$ implies a subtle correlation between successive AE hits measured at constant pressure rate. This seems to be borne out by the data, as presented in Figure 3. Note also that the burst distribution in time and amplitude can be very different from one system to another, as seen in Figure 3 by comparing system 1 to system 2: the distributions of bursts in time and amplitude are not universal; however, when they exist, on average they are correlated according to the logarithmic oscillations, whose mathematical structure is a universal property. For system 2, the log-periodic oscillations are less obvious but are indeed necessary to account for the still significative distorsion from a pure power law ($C = 0$ in Eq. (4)).

We now show that these results can be exploited to improve dramatically the prediction. Using this theory for prediction is also a crucial test to further establish its validity, since one could argue that a good fit with five adjustable parameters has too much freedom to provide a safe proof of any theory. The basic goal of any theory is in its predicting power, which we now test as follows. Using a given AE data set such as those presented previously, we first coarse-grain the data as in Figures 2 and 3. We then construct another data set by erasing all data above an upper pressure $p_{\max}$. This mimics an experiment performed under the same conditions but which would have stopped at the pressure $p_{\max}$ without reaching the pressure for global rupture. We then apply the non-linear fit to this truncated file and deduce $p_r^{\mathrm{predicted}}$ as one of the five variables of this fit. Figure 4 shows for system 1 with

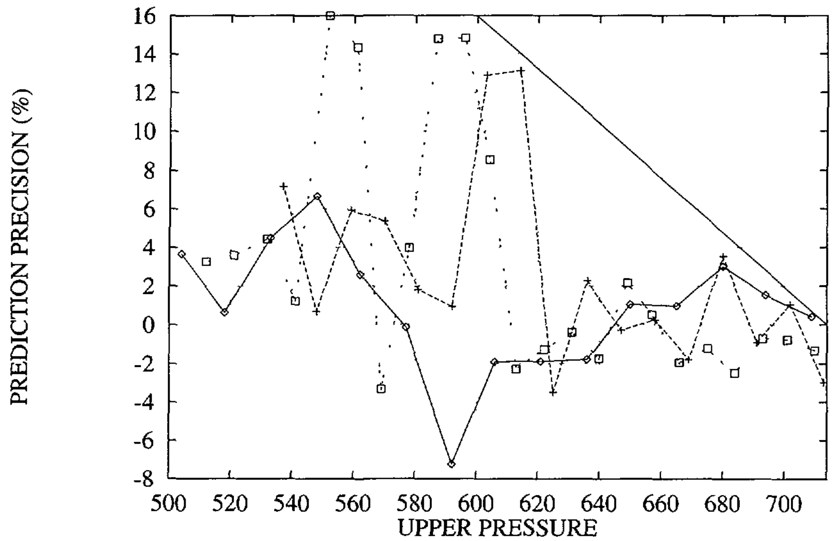

Fig. 4. — Relative precision of the prediction, namely $100(p_r - p_r^{\text{predicted}})/p_r$, as a function of $p_{\max}$ for system 1 with three different coarse-graining represented by the three different symbols, where $p_{\max}$ is the upper pressure taken into account. The straight line is given by $100(p_r - p_{\max})/p_r$ enabling a comparison between the precision of the prediction and the upper attained pressure.

three different coarse-graining (hence the three sets of symbols) the relative precision of the prediction, namely $100(p_r - p_r^{\text{predicted}})/p_r$ as a function of $p_{\max}$. Changing $p_{\max}$ allows us to explore the reliability of "long-term" prediction, i.e. far from rupture. Comparing different coarse-graining is important to assess the robustness and consistency of the prediction scheme. We observe that a precision better than 4% is obtained for $p_{\max} \geq 620$ bars, i.e. remarkably far below the rupture threshold, where use of a single oscillation in the data shown in Figure 3 (which has apparently little to do with the power law (1)), is sufficient to get the information on p_r. This remarkable success, in view of the relatively tiny amount of remaining AE data used in the fit, can be attributed to the very strong constraint imposed by the mathematical solution of equation (3), hence relies fundamentally on the physical picture of rupture as a critical point. For lower values ($p_{\max} < 620$ bars), the prediction deteriorates since lowering further $p_{\max}$ prevents from retrieving the information embedded in the first lobe of the data shown in Figure 3. Note that a similar procedure using only the asymptotic power law (1) yields extremely bad predictions as soon as $p_{\max}$ is lower than 680 bars. We have also tested our method "blind-folded", i e. on AE data obtained on systems which have not reached their rupture threshold. The comparison between our prediction and the actual pressure threshold reached in a subsequent experiment was of similar quality as shown in Figure 4. We have also tested different loading procedures; our results remain robust in all these cases.

In summary, our main point is that mechanical breakdown of heterogeneous media, instead of nucleating on a single crack as would be the case in a homogeneous system, involves collective phenomena. Idealised Ising nucleation models have recently been proven to exhibit a similar behavior [22]: for a range of parameters, nucleation of a single droplet leads to the decay of supersaturated vapor, whereas for other parameter combinations the coalescence of clusters arising from many separate nucleation events drives the phase transition. This mechanism has been confirmed by laboratory experiments and computer simulations. Exploiting these ideas, we have described, and tested on industrial systems, a new scaling theory of rupture viewed as a critical point endowed with a complex critical exponent, which allows one to get precise and reliable prediction of rupture thresholds from the analysis of AE data obtained on pressure load at constant pressure rates. Applications and extensions of these ideas for earthquakes will be presented elsewhere [23].

Acknowledgments

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