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Résumé.- Nous examinons la diffraction des neutrons thermiques par le réseau des vortex d'un supraconducteur de deuxième espèce. Nous montrons que l'ancrage des vortex non seulement introduit un élargissement des pics de Bragg mais aussi une composante diffuse dans la première zone de Brillouin qui, à champ élevé, est plus intense que la composante de Bragg.

Abstract.- The diffraction of neutrons from the imperfect vortex lattice of a type-II superconductor is investigated. It is found that flux pinning leads not only to a broadening of the Bragg reflections but also to diffuse scattering into the first Brillouin zone, which at high fields exceeds Bragg scattering.

In recent years there has been much effort to interpret the volume pinning force in type-II superconductors in terms of elementary pinning interactions between flux lines (FLs) and crystal lattice defects/1-2/. Up to now the statistical summation theories required for this interpretation are still unsatisfactory. It is therefore desirable to get more insight into the nature of these interactions. This may be achieved by neutron diffraction, which probes the magnetic field inside the superconductor. Flux pinning changes the diffraction pattern of the ideal flux line lattice (FLL) in two ways : the Bragg reflections are broadened/3/ and a finite scattering intensity occurs for scattering wavevectors in the first Brillouin zone (BZ)/4/. A relationship between the scattered intensity and the pinning interactions may be established in 5 steps.

1.- The differential scattering cross section for neutrons in Born approximation is related to the Fourier transform $\underline{\tilde{H}}(\underline{k})$ of the magnetic field $\underline{H}(\underline{r})$ inside the specimen by (in SI units)

$$d\sigma/d\Omega = (\mu_o \gamma_n / 4\phi_o)^2 |\underline{\tilde{H}}(\underline{k})|^2 \qquad (1)$$

where $\gamma_n = 1.91$, ϕ_o is the flux quantum, and $\underline{k}$ the scattering vector.

2. For a given FL displacement field with Fourier transform $\underline{\tilde{s}}(\underline{k})$, the Ginzburg-Landau (GL) theory yields for small FLL strains, for small FL tilting angles, and for $\underline{k}$ well inside the first BZ

$$\underline{\tilde{H}}(\underline{k}) = (\phi_o/\mu_o)(1+k^2/k_h^2)^{-1} i\underline{k} \times |\underline{\tilde{s}}(\underline{k}) \times \underline{\hat{z}}| \qquad (2)$$

where $\underline{\hat{z}}$ is the unit vector along the applied field, $k_h^2 \approx (1-b)/\lambda^2$, $b = B/B_{c2}$, and $\lambda = \kappa\xi$ is the field penetration depth. Equation(2) applies to $k < 0.7k_B$ where $k_B = (2b)^{1/2}/\xi$ is the BZ radius. An extension of (2) to larger k, which is required for a rigorous calculation of the profiles of Bragg reflections, has not been obtained yet for $0<b<1$. Inserting (2) in (1) we get for scattering into the first BZ

$$d\sigma/d\Omega = 0.23(1+k^2/k_h^2)^{-1}(|\underline{k}\underline{\tilde{s}}(\underline{k})|^2 + k_z^2|\underline{\tilde{s}}(\underline{k})|^2) \qquad (3)$$

3. As a next step we express the displacement field $\underline{\tilde{s}}(\underline{k})$ in terms of the pinning force field $\underline{\tilde{P}}(\underline{k}) = \underline{\underline{\tilde{\Phi}}}(\underline{k})\underline{\tilde{s}}(\underline{k})$ where $\underline{\underline{\tilde{\Phi}}}(\underline{k})$ is the elastic matrix of the FLL. The GL result for $k < 0.7\,k_B$ ("continuum approximation") and arbitrary b and κ is /5/

$$\underline{\underline{\tilde{\Phi}}}(k) = \frac{1}{n}\begin{pmatrix} c_{11}(k)k_x^2+c_{66}k_y^2+c_{44}(k)k_z^2+\alpha_L ; & [c_{11}(k)-c_{66}]k_xk_y \\ [c_{11}(k)-c_{66}]k_xk_y ; & c_{11}(k)k_y^2+c_{66}k_x^2+c_{44}(k)k_z^2+\alpha_L \end{pmatrix} \qquad (4)$$

where $n = B/\phi_o$, c_{66} is the shear modulus, and $c_{11}(k) \approx c_{11}(1+k^2/k_h^2)^{-1}(1+k^2/k_\psi^2)^{-1}$ and $c_{44}(k) \approx c_{44}(1+k^2/k_h^2)^{-}$ are the $\underline{k}$-dependent compressional and tilting moduli of the FLL, $k^2 = k_x^2+k_y^2+k_z^2$ and $k_\psi^2 = 2(1-b)/\xi^2$. The Labusch parameter α_L emerges from statistics of pinning forces/6/ and for consistency must be small $\alpha_L/c_{11} \ll k_B^2$ and $\alpha_L/c_{11} \ll k_h^2 \leq k_\psi^2$. Insertion in (3) of pinning forces requires the contribution to $\underline{\tilde{s}}(\underline{k})$ of the intrinsic FLL defects to be eliminated somehow. Fortunately, the elementary defects exhibit mere shear strains (point defects, edge dislocations) or tilt strains (screw dislocations), which all satisfy div $\underline{s} = \underline{k}\underline{s} = 0$ and thus do not contribute to (3) if $k_z = 0$; this can be achieved by choosing the incident neutron beam parallel to the applied field. In this scattering geometry we get simply

$$d\sigma(k_x,k_y,0)/d\Omega = 0.23n^2(1+k^2/k_\psi^2)^2(c_{11}k^2+\alpha_L)^{-2} |\underline{k}\underline{\tilde{P}}(k_x,k_y,0)|^2 \qquad (5)$$

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4. The pinning force field $\tilde{\underline{P}}(\underline{k})$ is related to the elementary pinning force fields $\tilde{\underline{P}}_i(\underline{k})$ of a large number, N, of weak pinning centers at positions $\underline{a}_i$ distributed at random by

$$|\underline{k}\tilde{\underline{P}}(\underline{k})|^2 = |\sum_i \underline{k}\tilde{\underline{P}}_i(\underline{k})|^2 = \sum_i |\underline{k}\tilde{\underline{P}}_i(\underline{k})|^2 \qquad (6)$$

The sum over the cross terms $\tilde{\underline{P}}_i\tilde{\underline{P}}_j$ vanishes because of destructive interference of phase factors $\exp\{i\underline{k}(\underline{a}_i-\underline{a}_j)\}$ belonging to pinning centers of the same type at different positions. This applies to a stiff FLL. Elasticity of the FLL leads to correlation of the $\tilde{\underline{P}}_i(\underline{k})$, since application of one force changes all FL positions and thus changes all other forces. This correlation is partly accounted for by introduction of α_L in (4) ; α_L removes the divergence of (5) at $\underline{k} = 0$. Besides this, the elastic response depresses the forces below the values obtained for a stiff and perfect FLL. This effect is neglected below.

5. The pinning forces may be derived from the perturbation

$$F_1 = \int d^3\underline{r}\{-\alpha|\Psi|^2+\tfrac{1}{2}\beta|\Psi|^4+\gamma|\nabla/i\kappa-\underline{A})\Psi|^2+\tfrac{1}{2}\chi H^2\} \qquad (7)$$

of the GL free energy by a method developed in/7/. The spatial function α, β, γ, and χ are small deviations from the mean ; for small pinning centers with diameter $<\xi$ they may be replaced by a delta function $\delta(\underline{r}-\underline{a}_i)$ with weight α_o, β_o, γ_o or χ_o. We discuss three special cases:

Short range forces.- Pinning centers with core interaction at $b < 0.3$, or magnetic interaction at $b < 1/2\kappa^2$, exert a force $\underline{P}_i$ only on the FL with position $\underline{R}_i$ closest to $\underline{a}_i$. This gives $\tilde{\underline{P}}_i(\underline{k})=\underline{P}_i\exp(-i\underline{k}\underline{R}_i)$ and

$$d\sigma/d\Omega = 0.23n^2(c_{11}k^2+\alpha_L)^{-2}k^2\ \frac{1}{2}\ \sum_i \underline{P}_i^2 \qquad (8)$$

In (8) the nonlocality parameters k_h and k_Ψ do not appear.

Long range forces.- Magnetic interaction at $b>>1/2\kappa^2$ yields

$$d\sigma/d\Omega=0.23N\chi_o^2n^2(c_{11}k^2+\alpha_L)^{-2}(1+k^2/k_\Psi^2)^2(1+k^2/k_h^2)^{-2}\, B^4/\mu_o^4 \qquad (9)$$

This is $\sim k^{-4}$ for $\alpha_L/c_{11}<<k^2<<k_\Psi^2$ and const. (k) for $k^2>>k_\Psi^2=k_B^2(1-b)/b$.

Core interaction at $b > 0.5$.- For $\alpha \neq 0$, $\beta = \gamma = \chi = 0$ we get

$$d\sigma/d\Omega= 0.20N\alpha_o^2n^2b^2(c_{11}k^2+\alpha_L)^{-2}k^4\{1+\tfrac{1}{4}[(1-b)k_B/bk + k/k_B]^2\} \qquad (10)$$

This is $\sim k^{-2}$ for $\alpha_L/c_{11}<<k^2<<k^4/2k_B$. In the large range $k_B(1-b)/2b<k<0.7\ k_B$ (10) reduces to

$$d\sigma/d\Omega \approx 0.20(8\pi^2)^{-1}N\alpha_o'^2\xi^2(2k^2-1)^{-2}= \text{const.}(k,b) \qquad (11)$$

In (11) we have used $c_{11}=B^2\partial H_a/\partial B$ and $\alpha_o=\alpha_o'\xi^3H_c^2\mu_o$ with $|\alpha_o'| \lesssim 1$. A similar result is obtained for $\gamma \neq 0$/4/.

The large constant scattering cross section (9) and (10) at large inductions is entirely due to the nonlocality of the elastic response of the FLL, in particular to the factor $(1+k^2/k_\Psi^2)^{-1}$ in $c_{11}(k)$. The factors $(1+k^2/k_h^2)^{-1}$ in the "magnetic response" (2) and in $c_{11}(k)$ cancel in the scattering cross section. At large inductions the intensity of small angle scattering exceeds that of the Bragg reflections, which vanishes as $(1-b)^2$. At still higher inductions FL trapping leads to a breakdown of the above theory and to $d\sigma/d\Omega = 0$ at $b = 1$. The derivation of our results applies to arbitrarily weak pinning and does not require a threshold to be exceeded as in theories of the volume pinning force. Small angle scattering of cold neutrons should thus yield insight into the pinning problem beyond that obtainable from measurements of the volume pinning force.

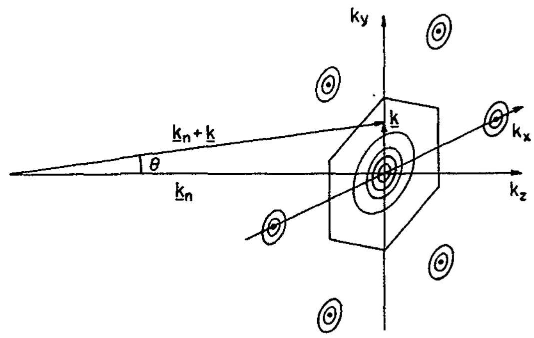

Fig. 1 : Longitudinal scattering geometry. Dots : reciprocal flux line lattice points, hexagon : first Brillouin zone, ellipses : lines of constant scattering intensity for $k_z = 0$, $\underline{k}_n$: wavevector of incident neutrons, $\underline{k}$: scattering vector, $|\underline{k}_n+\underline{k}|=|\underline{k}_n|\gg|\underline{k}|$ θ : scattering angle.

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