How copper contamination pulses shape the regime shifts of phytoplankton–zooplankton dynamics?

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Abstract

In this study, we consider the effects of impulsive copper contamination of the phytoplankton−zooplankton dynamics. We use the model on interactions between algae and Daphnia with deterministic and stochastic impulse copper contamination. In low environmental copper concentration ($Cu_{cst} < 4.4\mu g L^{-1}$) our analysis show that, deterministic and stochastic pulses may promote the persistence of Daphnia and algae populations unlike the absence of pulses. We show that deterministic and stochastic pulses accelerate deficiency and toxicity processes, leading to the extinction of all populations, in high ($Cu_{cst} > 28\mu g L^{-1}$) minimal of copper concentrations. In intermediate concentrations, deterministic and stochastic pulses may transform population dynamics in complex oscillations. Bifurcation diagram was computed to illustrate the different type observed dynamics in an environment with pulses of contamination. Depending on minimum copper concentration in the environment, this bifurcation diagram highlighted, the resilience or the regime shifts of the system in occurrence of pulse contamination. Our study may contribute to the prevalence of underestimation of extinction risk or population regime shifts from random fluctuations of pollution in real

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ecosystems.

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1. Introduction

The field of nonlinear science has seen a growing interest for complex dynamics [23, 18, 19], particularly in marine ecosystems, which are a subset of earth's aquatic ecosystems [1]. They include lakes and ponds, rivers, streams, springs, and wetlands. They can also be contrasted with marine ecosystems, which have a larger salt content. Freshwater habitats can be classified by different factors, including temperature, light penetration, and vegetation. Freshwater ecosystems can be divided into lentic ecosystems (still water) and lotic ecosystems (flowing water). It is true that the populations of these ecosystems need some external nutrients to live, such as copper concentration, which is an essential metal, toxix at low and high dose. Its great use in dispersive uses and its know toxicity for many organisations leads naturally to assess the risk to aquatic ecosystems associated with contamination by this metal. Thus, several mathematical models of ecosystems have been proposed and implemented for understanding and controlling the impact of the contamination of those nutrients[2, 3, 4, 6].

Many models including nutrient or copper concentration has been studied in [6, 7, 8, 9, 10]. Hallam [12] studied stability and persistence properties of a family of nutrient-controlled plankton models and obtained necessary and sufficient conditions for persistence. Gard [14] studied a nutrient-phytoplankton-zooplankton (NPZ) model with generalized functional response and obtained sharper criteria for persistence. A phytoplankton-zooplankton model was studied by Steffen et al. [16]; local and global behavior of the model were obtained.

More recently, Prosnier et al.[6] proposed the model for predicting the continuous effects of pollution at the community level such as effects of copper on phytoplankton-zooplankton interactions, which is a challenge difficult because of the complex impacts of ecosystem dynamics and properties. They showed that: i) low and high copper concentrations cause deficiency and toxicity, respectively, leading to the extinction of all populations; for less extreme concentrations, only the consumer population becomes extinct. The two populations survive with intermediate concentrations; ii) when population dynamics present oscillations, copper has a stabilizing effect and reduces or suppresses oscillations; iii) copper, on account of its stabilizing effect, opposes the destabilizing effect of nutrient enrichment. Despite the growing interest in environmental pulse contamination [5, 13, 15], the effects of pulsed toxicants and the subsequent recovery of the population remain problematic. In addition, Folke *et al.* [11] have highlighted the importance, in pulse contaminated ecosystem, of possible irreversible changes in ecosystems induced by regime shifts. In this paper, we propose to analyze effects of impulsion copper contamination on interactions between algae and Daphnia. We will assume in the analysis that the copper contamination in the population of ecosystem is not continuous, this contamination will occur after a time period σ and will happen again after a number of days $n \times \sigma$. n is an integer for a deterministic copper contamination, and a random number for a stochastic contamination. Effects of random pulse amplitude are also discuss in the paper. The study of effects of impulsion copper contamination on interactions between algae and Daphnia is motivated by several reasons: i) the model with impulsive contamination are much closer to reality because it does not exist in the environment of ecosystems where contamination is continuing. ii) in degraded ecosystems of copper; copper intake in the form of pulses will maintain the coexistence of the algae and Daphnia populations, our model will make recommendations to managers of degraded ecosystems.

The paper is organized as follows: first, we present the model on interactions between algae and Daphnia with continue, deterministic and stochastic impulse copper contamination in section 2. In section 3, we present through numerical simulations the results and discussions. In this part, we show the effects of minimal pulse values on species dynamics and survival of species. The effects of stochastic copper pulses will also analyze. This section end with the bifurcation structures which reveal the effect of changes in a particular system parameter on Daphnia-Scenedesmus interaction. In Section 3, some conclusions and discussions complete the paper.

2. Materials and methods

2.1. Model with continue copper contaminations

The model consider in our analysis is based to a simple freshwater ecosystem consisting of two compartments, phytoplankton and zooplankton, with the genera Scenedesmus and Daphnia chosen as model organisms for these compartments. In ref [6], the authors develop the model follows with continuous copper contamination on interactions between algae and Daphnia. Let us summarize the modelisation here: the Scenedesmus-Daphnia interaction without copper pollution is described through the Rosenzweig-MacArthur model [17] based on logistic growth for Scenedesmus and a type II functional response for Daphnia [20, 21, 22], reads:

$$\begin{aligned} \frac{dS}{dt} &= r \times S \times \left(1 - \frac{S}{K}\right) - \frac{I_{max} \times S \times D}{S+h} \\ \frac{dD}{dt} &= \left(e \times \frac{I_{max} \times S}{S+h} - m\right) \times D \end{aligned} \tag{1}$$

where S and D are Scenedesmus and Daphnia densities $(mgCL^{-1})$,respectively, r Scenedesmus intrinsic rate of natural increase (d^{-1}), K Scenedesmus carrying capacity $(mgCL^{-1})$, I_{max} the maximum take rate of the Daphnia (d^{-1}), h the half-saturation constant of Daphnia (mgC/L), e the Daphnia conversion efficiency, and m Daphnia mortality rate (d^{-1}). Parameter values derived from the literature are given in Table 1.

To analyze the effects of copper on organisms, the authors showed that the internal copper concentrations for Scenedesmus (C_S) and Daphnia (C_D) as a function of external concentration (Cu) are as follows [6]:

$$C_S(Cu) = \left(\frac{Cu \times k_{mS}}{Cu + k_{cS}}\right) \times \frac{1}{K_{eS}} \tag{2}$$

$$C_D(Cu) = \left(\frac{Cu \times k_{mD}}{Cu + k_{cD}} + e \times \frac{I_{max} \times S}{S+h} \times C_S\right) \times \frac{1}{k_{eD}} \tag{3}$$

where k_{mS} and k_{mD} are the maximal ingestion rates $(\mu g g^{-1} d^{-1})$ of Scenedesmus and Daphnia, respectively, k_{cS} and k_{cD} their half-saturation constants $(\mu g L^{-1})$, and k_{eS} and k_{eD} their constant loss rates (d^{-1}).

The effects of copper on organism can be understood if one represent copper dose-response relationships by a sigmoid curve that captures only the effect of copper as a pollutant at high concentration. For a comprehensive analysis of the effects of copper in the population of phytoplankton and zooplankton, we note that this population needs a small quantity of copper for survival. Thus, one introduce the following asymmetric double sigmoid

Parameters	**Descriptions**	**Values**	**Units**
Population's dynamics			
r	Scenedesmus intrinsic rate of natural increase	1.2	d^{-1}
K	Range of Scenedesmus carrying capacity	0.1-5	$mgCL^{-1}$
I_{max}	Maximum intake rate of the Daphnia	1.8	d^{-1}
h	Half-saturation constant of Daphnia	0.164	$mgCL^{-1}$
e	Daphnia conversion efficiency	0.6	-
m	Daphnia mortality rate	0.35	d^{-1}
Copper-internal concentration			
Cu	Range of external copper concentration	0-100	μgL^{-1}
k_{ms}	Scenedesmus maximal intake rate	20	$\mu gg^{-1}d^{-1}$
k_{mD}	Daphnia maximal intake rate	15	$\mu gg^{-1}d^{-1}$
k_{cS}	Scenedesmus half-saturation constant	6	μgL^{-1}
k_{cD}	Daphnia half-saturation constant	7	μgL^{-1}
k_{eS}	Scenedesmus constant loss rate	1	μgd^{-1}
k_{eD}	Daphnia constant loss rate	1	μgd^{-1}
Copper-effects			
ν_r	Scenedesmus growth's deficiency EC_{50}	4	μgL^{-1}
u_r	Scenedesmus growth's toxicity EC_{50}	50	μgL^{-1}
d_r	Copper effect on Scenedesmus growth	5	-
b_r	Copper effect on Scenedesmus growth	2	-
ν_p	Daphnia predation's deficiency EC_{50}	5	μgL^{-1}
u_p	Daphnia predation's toxicity EC_{50}	16.8	μgL^{-1}
d_p	Copper effect on Daphnia predation	5	-
b_p	Copper effect on Daphnia predation	1	-
$LD_{50} - Daphnia$	Daphnia LD_{50}	30	μgL^{-1}
p_m	Copper response coefficient for Daphnia mortality	0.021	μgL^{-1}

Table 1: Value of model parameters used for numerical analyses, [6].

function with two thresholds, deficiency and toxicity [6]:

$$\begin{aligned} Cu_x(Cu) &= (a+c) - \frac{1}{2} \times (a-e) \times \tanh\left(d_x\left(C_x(Cu) - C_x(\nu_x)\right)\right) \\ &= +\frac{1}{2} \times (a-c) \times \tanh\left(b_x\left(C_x(Cu) - C_x(u_x)\right)\right) \end{aligned} \tag{4}$$

where Cu_x is the effect of copper on parameter x, $(a+c)$ the minimal value of the effect, $(a-e)$ the amplitude of the effect, ν the lower E_{C50}(deficiency) and u the higher E_{C50}(toxicity), and d and b the lower and higher slopes of the curve, respectively (see below for explanations). Let us recall through equation (4) the expression of the governed equation use to analyse the effect of copper on Scenedesmus growth rate, r, and the effect of copper on predation, respectively. The copper effect, Cu_r on Scenedesmus range from -1, low copper concentration, to +1, at high copper concentration. Thus, $(a+c) = -1$ and $(a-e) = -2$, so $a = -1.5$ and $c = e = 0.5$. One obtains from equation (4):

$$Cu_r(Cu) = -1 + \tanh\left(d_r\left(C_{S(Cu)} - C_{S(\nu_r)}\right)\right) - \tanh\left(b_r\left(C_{S(Cu)} - C_{S(u_r)}\right)\right). \tag{5}$$

While the effect of copper on predation, Cu_p, ranges between 1 and 0, that is, between no effect on predation at inter-mediate concentrations and total inhibition of predation at low and high concentrations. Thus, $(a+c) = 0$ and $(a-e) = -1$, so $a = -0.5$ and $c = e = 0.5$. We thus obtain from (4):

$$Cu_p(Cu) = \frac{1}{2}\tanh\left(d_p\left(C_D(Cu) - C_D(\nu_d)\right)\right) - \frac{1}{2}\tanh\left(b_p\left(C_D(Cu) - C_D(u_p)\right)\right) \tag{6}$$

Because there is no hormetic effect of copper on Daphnia mortality but only a negative effect, we model a linear effect on this parameter as follows:

$$Cu_m = 1 + p_m \times C_D(Cu) \tag{7}$$

where p_m is a copper response coefficient. Finally, introducing the copper effects captured in Eqs. (4)-(7) into model (1) leads to the following equations:

$$\begin{aligned} \frac{dS}{dt} &= r \times Cu_r \times S \times \left(1 - \frac{S}{K}\right) - \frac{I_{max} \times S \times D}{S+h} \times Cu_p \\ \frac{dD}{dt} &= \left(e \times Cu_p \times \frac{I_{max} \times S}{S+h} - m \times Cu_m\right) \times D \end{aligned} \tag{8}$$

We make the following realistic assumption taking into account that this population needs a small quantity of copper for survival and high copper concentration induces negative effects on daphnia and algae populations. These copper effects are represented by a sigmoid function [6].

2.2. Model with deterministic copper pulse input

In this section, we assume that the copper contamination of organisms is not continue but as deterministic daily pulses given by the following function:

$$Cu(t) = Cu_{cst} + \sum_{j=1}^{N} b\Pi\left(\frac{t - t_j}{\sigma}\right) \tag{9}$$

where $\sigma = 1, 2, 3, ...$ is the day duration contamination pulses; Cu_{cst} is the constant copper concentration of an environment without copper pulse; N is the number of pulses; t_j $(j = 1, 2, 3, ..., N)$, the day of pulses in regular time intervals; $\Pi(x)$ is the rectangular function and b measure the amplitude of the pulses. Therefore the cooper function effects, expressed in equation (5) and equation (6), depend also on time as follows $Cu_r\left(Cu(t)\right)$, $Cu_p\left(Cu(t)\right)$. Considering deterministic copper pulses, figure 1 shows respectively the daily copper variation (fig 1(A)), the daily copper effects on algae growth (fig 1(B)) and the daily copper effects on predation (fig 1(B)).

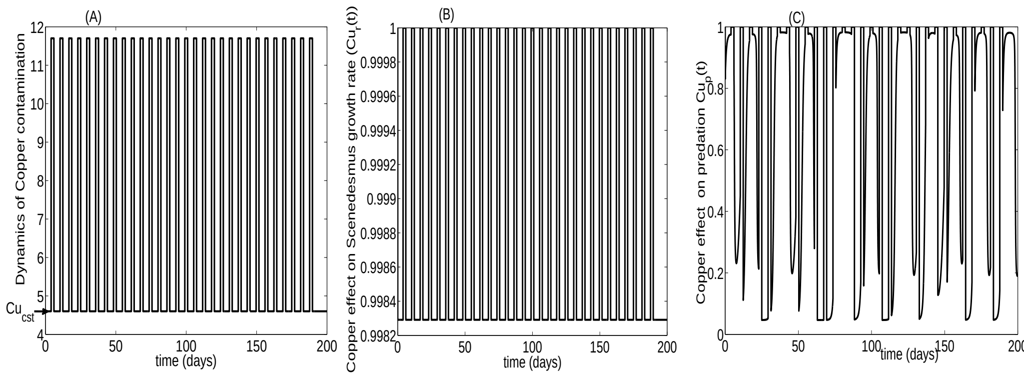

Fig. 1: (A) represents the daily cooper concentration $Cu(t)$ from equation 9 with $\sigma = 2$ days, $b = 7\ \mu g L^{-1}$, $Cu_{cst} = 4.6\ \mu g L^{-1}$. (B) and (C) are the correspond measured effects on growth $Cu_r\left(Cu(t)\right)$, time of pulse events $t_j = 7 \times j$ days and predation $Cu_p\left(Cu(t)\right)$.

2.3. Model with stochastic copper pulse input

In this section we consider that the copper contamination of organisms follows stochastic daily pulses, expressed as:

$$Cu(t) = Cu_{cst} + \sum_{j=1}^{N} b_j \exp\left(-\frac{1}{2}\frac{(t-t_j)^2}{\sigma^2}\right)\Pi\left(\frac{t-t_j}{\sigma}\right) \qquad (10)$$

where $\sigma = 1, 2, 3, ...$ is the day duration contamination pulses; Cu_{cst} is the constant copper concentration in an environment without copper pulse; N is the number of pulses; $t_j (j = 1, 2, 3, ..., N)$, the day of pulses in regular time intervals; $\Pi(x)$ is the rectangular function; $b_j \quad (j = 1, 2, .., N)$ measure the random amplitude of the pulses in the time; $e^{-\frac{1}{2}\left(\frac{t-t_j}{\sigma}\right)^2}$ measure a Gaussian repetition of a pulse around the date t_j. Therefore the cooper function effects, expressed in equation (5) and equation (6), depend also on time as follows $Cu_r\left(Cu(t)\right)$, $Cu_p\left(Cu(t)\right)$. Considering stochastic copper pulses, figure 1 shows respectively the daily copper variation (fig 2(A)), the daily copper effects on algae growth (fig 2(B)) and the daily copper effects on predation (fig 2(B)).

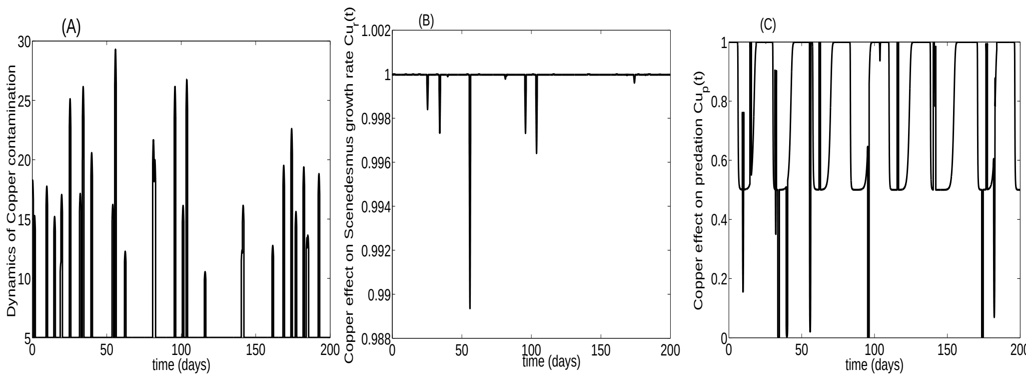

Fig. 2: (A) represents the daily cooper concentration $Cu(t)$ from equation 10 with $\sigma = 1$ days, b follows a χ^2 distribution with five degree of freedom, $Cu_{cst} = 5\ \mu g L^{-1}$, the number of events $N = 30$, time of events t_j follows a uniform distribution in $[0\ ,\ 200\ days]$. (B) and (C) are the correspond measured effects on growth $Cu_r\left(Cu(t)\right)$ and predation $Cu_p\left(Cu(t)\right)$.

3. Results

3.1. Effects of deterministic copper pulse

3.1.1. Effects of minimal pulse values on species dynamics

Compared to Model 1, for which the Daphnia population turns off to constant low minimum concentrations of copper ($Cu_{cst} < 4.4\ \mu gL^{-1}$), model 8, with periodic pulses of one day duration a constant intensity, lead to the persistence of daphnia population when constant copper concentrations is low. If fact, alga Scenedesmus and daphnia population follow oscillatory dynamics when Cu_{cst} is closed to 4.36 μgL^{-1}. Similarly, the episodic pulses induce the same effects of daphnia and alga persistence for very high constant concentrations ($Cu_{cst} \geq 30\ \mu gL^{-1}$) causing the extinction of daphnia. The model 1 simulations, in figure 3(A), illustrates the extinction of Daphnia in the absence of pulses, when the copper concentration is constant and is $Cu = 4.38\ \mu gL^{-1}$. Figure 3(B) represents daphnia and Scenedesmus oscillation dynamics in deterministic copper pulses having an intensity $b = 5\ \mu gL^{-1}$, one day duration and minimal value $Cu_{cst} = 4.38\ \mu gL^{-1}$.

Model 1 exhibits periodic oscillations (figure 3 (C)), when constant copper concentration reaches a certain threshold ($Cu \approx 18.62\ \mu gL^{-1}$). However, an incorporation into this model of regular pulse of copper contamination can induce disturbance of daphnia and alga scenedesmus population dynamics. Thus, using copper pulses having one day duration and an intensity $b = 5\ \mu gL^{-1}$ can lead to a doubling period oscillations ($Cu_{cst} = 18.62$) or aperiodic oscillations ($Cu_{cst} = 18.49\ \mu gL^{-1}$) of Daphnia and alga Scenedesmus densities (figure 3 (D)).

3.1.2. Pulse duration and survival of species

Unlike the absence of pulses or the presence of pulses with one day duration, the pulses with three days duration maintain the coexistence of species (see fig 4 (A)), when the minimum concentration of copper is low or close to zero ($0 \leq Cu_{cst} \leq 4.36\ \mu gL^{-1}$). In general, in case of low concentrations of copper in the environment, regular episodic pulses better maintain the coexistence of people that the absence of pulses. However on very high minimum concentrations ($Cu_{cst} > 25\ \mu gL^{-1}$), the pulse accelerate the extinction of daphnia. Indeed for a $Cu = 28.3\ \mu gL^{-1}$ contamination in the absence of pulse, the daphnia population decline after 28 days, whereas with pulses ($Cu_{cst} = 28.3\ \mu gL^{-1}$) daphnia extinction is observed after 12 days (see fig 4 (B) (C)).

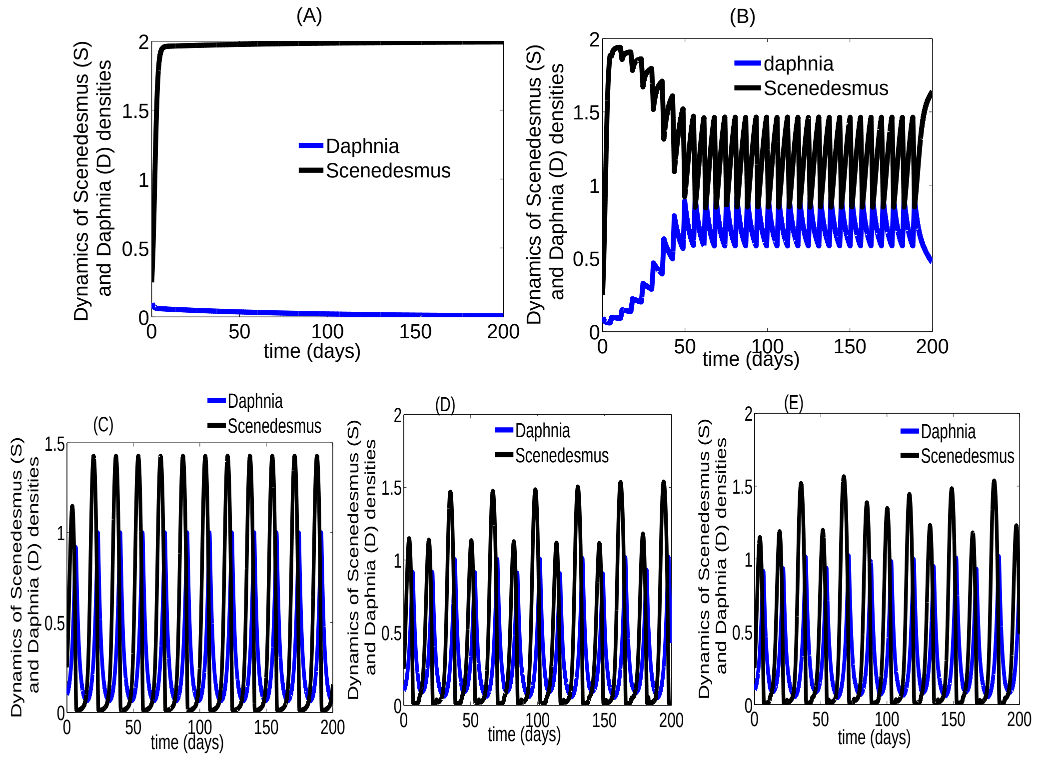

Fig. 3: Daphnia and alga scenedesmus dynamics, parameters of models are fixed as in table 1. (A) and (C) are model 1 output having constant copper concentration $Cu = 4.38$ and $Cu = 18.62$ respectively. (B, (D) and (E) are model 8 output having deterministic pulses, constant pulse intensity $b = 5\ \mu g L^{-1}$, pulse duration $\sigma = 1$ day and minimal pulse value respectively $Cu_{cst} = 4.38\ \mu g L^{-1}$ $Cu_{cst} = 18.62\ \mu g L^{-1}$ and $Cu_{cst} = 18.49\ \mu g L^{-1}$.

3.2. Effects of stochastic copper pulses

3.2.1. Type of dynamics in stochastic copper pulses

In this part, the event times of copper pulses and the pulse intensities are randomly generated. Figure 5 (A) is a phase portrait of model 1 with parameter $Cu = 4.415\ \mu g L^{-1}$. Phase portrait of model 8 with parameter $Cu_{cst} = 4.415\ \mu g L^{-1}$ is showed in figure 5 (B) (respectively (C)) by setting a deterministic time $t_j = 7 \times j\ days$ of pulse events and random pulse intensity b_j (respectively a random time t_j of pulse events and deterministic pulse intensity $b = 5\ \mu g L^{-1}$). Thus, the random copper pulses can significantly disrupt the dynamics of Daphnia and alga Scenedesmus, by changing a stable equilibrium convergence (fig. 5 (A)) to complex dynamics (fig. 5 (B)

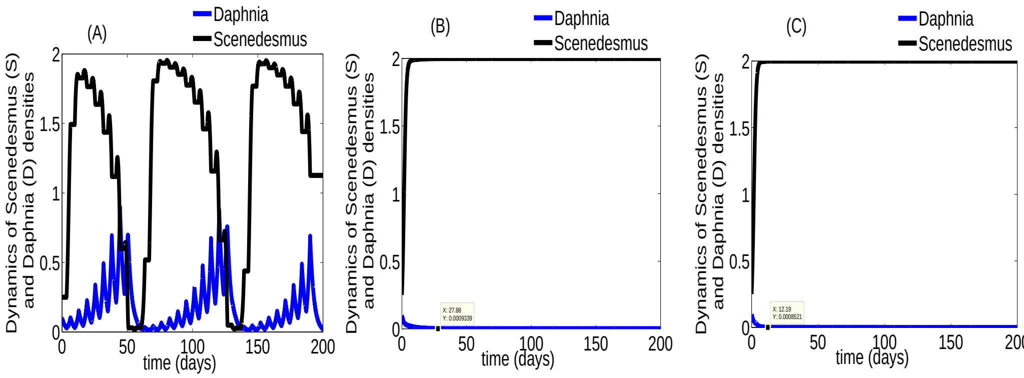

Fig. 4: Daphnia and alga scenedesmus dynamics from model 8 with parameters fixed as in table 1, $\sigma = 3$ days pulse duration and constant pulse intensity $b = 5\ \mu g L^{-1}$. (A) is population oscillation dyanamics when minimal pulse value $Cu_{cst} = 0$. (B) and (C) represents daphnia extinction population from model 8 model 1 respectively, when minimal pulse value $Cu_{cst} = 28.3\ \mu g L^{-1}$ and $Cu = 28.3\ \mu g L^{-1}$.

and (C)). According to our simulation, these complex or chaotic dynamics generally result from single random factor (random time of events or random pulse intensity as in fig 5 (B) and C)) or a combination of theses two factors (see fig 5 (D), (E) and (F)). Observed complex or chaotic dynamics (fig 5 (F)) are slightly modified when we have a random time of pulse events with a fixed pulse intensity (fig 5 (E)). But, deterministic time of pulse event with random pulse intensity change more the complex dynamics resulting from theses to random factors. These types of complex or chaotic dynamics are not observable with the model without pulse of contamination.

As in the deterministic case, stochastic contamination pulses can maintain the coexistence of two populations when the minimum amount of copper (Cu_{cst}) in the environment is low. Furthermore the pulse duration σ play an important role of population persistence in low environmental copper concentration. Indeed the minimum amount of copper Cu_{cst} for the survival and population persistence, decreases with increasing duration of pulses. However, for very high contaminated environment $Cu_{cst} > 28\ \mu g L^{-1}$), our stochastic pulse simulations show an extinction of Daphnia population.

3.2.2. Bifurcation structures

We summarize qualitative aspect of regime shift by using bifurcation diagrams that graphically reveal the effect of changes in a particular system

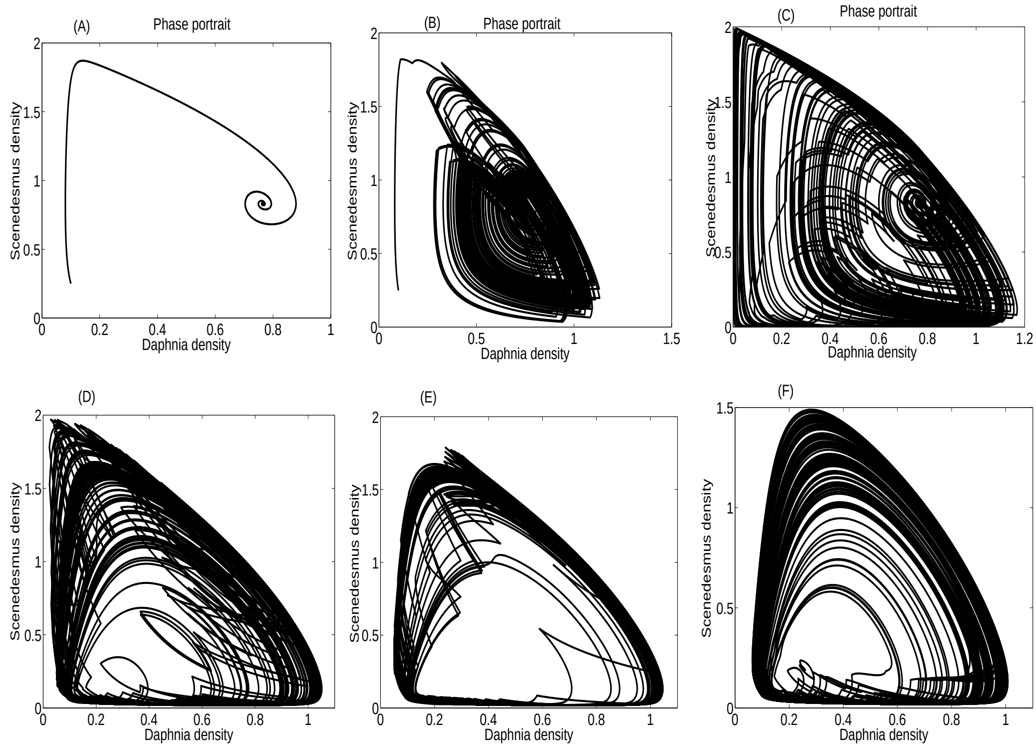

Fig. 5: Daphnia and alga scenedesmus dynamics from model 8 with parameters fixed as in table 1, $\sigma = 3$ days pulse duration and constant pulse intensity $b = 5$ $\mu g L^{-1}$. (A) is population oscillation dyanamics when minimal pulse value $Cu_{cst} = 0$. (B) and (C) represents daphnia extinction population from model 8 model 1 respectively, when minimal pulse value $Cu_{cst} = 28.3$ $\mu g L^{-1}$ and $Cu = 28.3$ $\mu g L^{-1}$.

parameter on Daphnia-Scenedesmus interaction. The bifurcation diagram were produced computing Poincare map with matlab. We focus on bifurcations in the population dynamics in response to changes in the minimal copper pulse Cu_{cst} of a system having random pulses of copper contamination in time and in intensity. The bifurcation diagram, show in figure 6, traces the steady-state responses of the daphnia density and the extrema of the oscillations, when cycles are observed, while identifying stable and unstable regions. Bifurcations in this system occur when a steady state gives way to oscillations or when a small amplitude cycles give way to a large amplitude cycles. As it appears on figure 6, different types of bifurcations take place. As the minimal copper pulse Cu_{cst} increases from $4.4\mu g L^{-1}$, the Daphnia-Scenedesmus system exhibit complex oscillatory dynamics with multiple amplitude cycle until $Cu_{cst} = 5.15\mu g L^{-1}$ where the complex behavior bifurcates into the periodic oscillatory state. Then at $Cu_{cst} = 14.0\mu g L^{-1}$, a tiny multiperiodic transition appears and the system passes into another complex oscillatory state. These complex dynamics from model 8 can not be observed with model 1. As Cu_{cst} increases further, a period transition takes place at $Cu_{cst} = 27.6\mu g L^{-1}$ and the system exhibit periodic oscillatory state until $Cu_{cst} =$ 28.0 $\mu g L^{-1}$. These periodic oscillations and stable equilibria are observed in whatever in whatever way random copper pulses are generated. Daphnia population decline when $Cu_{cst} > 28.0$ $\mu g L^{-1}$.

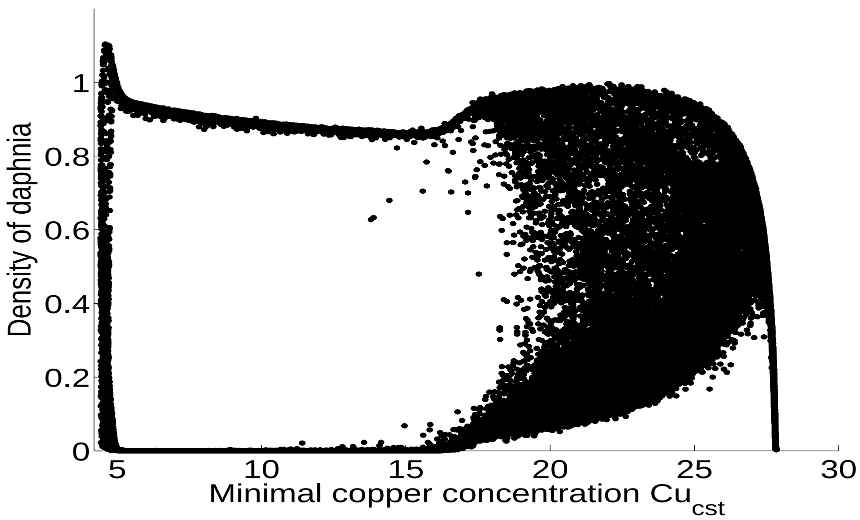

Fig. 6: Daphnia and alga scenedesmus dynamics from model 8 with parameters fixed as in table 1, $\sigma = 3$ days pulse duration and constant pulse intensity $b = 5$ $\mu g L^{-1}$. (A) is population oscillation dyanamics when minimal pulse value $Cu_{cst} = 0$. (B) and (C) represents daphnia extinction population from model 8 model 1 respectively, when minimal pulse value $Cu_{cst} = 28.3$ $\mu g L^{-1}$ and $Cu = 28.3$ $\mu g L^{-1}$.

4. Conclusion

In this paper, we have investigated the dynamics of the phytoplankton-zooplankton interactions model with deterministic and stochastic impulsion copper contamination. Our contributions are as follows:

- We consider the effects of deterministic impulsive copper contamination on a simple freshwater prey-predator ecosystem. The effects of the impulses copper contamination on the population are considered in detail. Our results reveal that the impulses copper contamination has great impacts on the model. Our analyses show that depending on the nutrient status of the system, impulsion copper pollution may stabilize the prey-predator interaction, this stabilization do not lead, as in the case of continuous contamination, to predator extinction if copper concentration increases further but to the survival of the prey and predator population.

- The effects of stochastic impulsive copper contamination are also considered. Our analysis highlights the existence of more complex dynamics depending on the random effect of the following parameters: event times, pulses amplitudes and pulse duration

Our Simulations based on model 8 suggest that population temporal behaviors after copper pulses of contamination can lead to complex dynamics. An combination of high ($14.0\ \mu g L^{-1} \leq Cu_{cst} \leq 27.6\ \mu g L^{-1}$) or low ($4.4\ \mu g L^{-1} \leq Cu_{cst} \leq 5.15\ \mu g L^{-1}$) copper minimal concentration with random pulses may lead to a long-term alteration of populations dynamics or regime shifts. In particular, and that not only the pulse random amplitude but also the time events of pulses and pulses duration are important factor for understanding population regime shifts. These regime shifts reveals the vulnerability and a loss of system resilience for minimal copper concentrations in these ranges of values ($14.0\ \mu g L^{-1} \leq Cu_{cst} \leq 27.6\ \mu g L^{-1}$ or $4.4\ \mu g L^{-1} \leq Cu_{cst} \leq 5.15\ \mu g L^{-1}$). Note also that the copper pulses have the beneficial effect of maintaining the coexistence of species when the minimum concentration is low. In random copper pulse environment, the dynamics of Daphnia-Scenedesmus population become oscillatory when the minimal copper pulse Cu_{cst} is in $]5.15\ \mu g L^{-1}\ ,\ 14.0\ \mu g L^{-1}[$, meaning that system is resilient for this Cu_{cst} values. This analysis allows to characterize the conditions of the resilience or the regime shirts systems and also provides a tool to improve the management of disturbed ecosystems.

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