NEUTRINO IDENTITY

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Résumé. — La théorie habituelle des interactions faibles contient une hypothèse simple, mais riche en conséquences, sur l'identité des neutrinos : il s'agit de la conservation indépendante des nombres quantiques muonique et électronique. Dans ce travail nous présentons une discussion critique des expériences qui pourraient tester cette hypothèse.

Abstract. — The conventional model of weak interactions incorporates a simple yet far reaching hypothesis concerning neutrino identity : the independent conservation of muon number and electron number. In this work, we argue that this hypothesis can and should be tested experimentally.

Certainly, the conventional phenomenological model of weak interactions is one of the finest triumphs of modern physics. A vast amount of experimental data is explained and correlated in terms of only two fundamental parameters, the coupling strength G and the Cabibbo angle θ. So successful is this model that many of its predictions, although they are not yet experimentally tested, are often tacitly — and, I think, unfortunately — assumed to be true. We shall speak in particular to the question of what are the neutrino states participating in leptonic and semi-leptonic phenomena : are the neutrinos associated with strangeness-conserving semi-leptonic processes the same as those associated with strangeness-changing processes ? How are these neutrinos related to those participating in muon decay and in diagonal leptonic processes ? The conventional model gives unambiguous answers to those questions. This talk is simply a plea to the experimentalist to be critical and not to assume that these answers must be true. Only experiment can decide the questions of neutrino identity, and confirm or deny our simple picture.

We first consider semi-leptonic processes. We do not question the V-A form of the theory, nor the chiral structure of lepton currents. We assume the overall conservation of lepton number, but not the independent conservation of muon number and electron number. Thus, we envisage the following form for the phenomenological Lagrangian :

$$G \cos \theta H_0^\lambda \{ \bar{e}\gamma_\lambda(1 + \gamma_5) \nu_1 + \bar{\mu}\gamma_\lambda(1 + \gamma_5) \nu_3 \} + \\ + G \sin \theta H_1^\lambda \{ \bar{e}\gamma_\lambda(1 + \gamma_5) \nu_2 + \bar{\mu}\gamma_\lambda(1 + \gamma_5) \nu_4 \} . \quad (1)$$

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Here, $H_0(H_1)$ is the usual normalized strangeness-conserving (changing) hadronic current. But, we have allowed for the possibility of four distinct neutrino states.

For the sake of orientation, we may consider three extreme possibilities. In a one-neutrino theory we would have

$$\nu_1 = \nu_2 = \nu_3 = \nu_4 . \quad (2a)$$

Another possibility is to make each neutrino distinct, and orthogonal to the others,

$$< \nu_i \mid \nu_j > = 0 \quad \text{for } i \neq j . \quad (2b)$$

Or, following the conventional theory, we could put

$$\nu_1 = \nu_2, \nu_3 = \nu_4 \\ < \nu_1 \mid \nu_3 > = 0 . \quad (2c)$$

Of these three simple possibilities, only the third is compatible with experiment [1].

But, it is clear that there is an infinite variety of other possibilities. In general, we must consider the 4×4 tableau of dot products of the various neutrino states

$$< \nu_i \mid \nu_j > = M_{ij} = \begin{bmatrix} 1 & a & c & d \\ \bar{a} & 1 & e & f \\ \bar{c} & \bar{e} & 1 & b \\ \bar{d} & \bar{f} & \bar{b} & 1 \end{bmatrix} . \quad (3)$$

In any presently conceivable experiment, it is only the absolute magnitudes of the six off-diagonal entries of M that are measurable. However, in a general model, these six parameters can each assume any value between zero and one. Each of these parameters has a direct experimental significance ; e. g., $| b |^2$ is determined by observing the rate of $\Delta Y = 1$ muon events produced by neutrinos from π decay, and $| d |^2$ is determined by observing the rate of $\Delta Y = 0$ electron events produced by neutrinos from $K_{\mu\nu}$ decays, etc...

In Table I we enumerate the predicted values of these parameters in the conventional model (2*c*), and my *guess* as to the experimental limits that may be extracted from available data. Observe that the extent of the confirmation of the conventional model is quite modest and that the possibility that a more exotic model applies cannot be ruled out. Let us hope that these limits improve with the next generation of neutrino experiments.

TABLE I

	Usual theory	Present limit
$\mid a \mid$	1	None
$\mid b \mid$	1	$\geqslant .50$
$\mid c \mid$	0	$\leqslant .15$
$\mid d \mid$	0	$\leqslant .25$
$\mid e \mid$	0	$\leqslant .75$
$\mid f \mid$	0	None

The next question we consider concerns the identity of neutrinos produced in muon decay and related leptonic phenomena. For simplicity, let us assume that the conventional theory is satisfied for semi-leptonic processes. Otherwise, the analysis is more complicated, and experimental results on leptonic processes would depend on what fraction of the neutrino spectrum originated from pions or kaons (an interesting possibility, which we do not mean to exclude).

We envisage the following phenomenological Lagrangian

$$G(\cos\theta H_0^\lambda + \sin\theta H_1^\lambda) \times$$
$$\times \{ \bar{e}\gamma_\lambda(1+\gamma_5)\nu + \bar{\mu}\gamma_\lambda(1+\gamma_5)\nu' \} +$$
$$+ g_1 \{ \bar{e}\gamma^\lambda(1+\gamma_5)\nu \} \{ \bar{\nu}'\gamma_\lambda(1+\gamma_5)\mu \}$$
$$+ g_2 \{ \bar{e}\gamma^\lambda(1+\gamma_5)\nu' \} \{ \bar{\nu}'\gamma_\lambda(1+\gamma_5)\mu \}$$
$$+ g_3 \{ \bar{e}\gamma^\lambda(1+\gamma_5)\nu \} \{ \bar{\nu}\gamma_\lambda(1+\gamma_5)\mu \}$$
$$+ g_4 \{ \bar{e}\gamma^\lambda(1+\gamma_5)\nu' \} \{ \bar{\nu}\gamma_\lambda(1+\gamma_5)\mu \} . \qquad (4)$$

We have included terms corresponding to every possible assignment of neutrino identity, assuming only that the same neutrino states participate in semi-leptonic and purely leptonic processes. In the conventional model, we must put $g_1 = G$ and $g_2 = g_3 = g_4 = 0$. Note that the terms involving g_2 and g_3 correspond to violations of muon number (or electron number) of unity, while the term involving g_4 corresponds to a violation of this quantum number by two. What appears as muon decay is any of the four processes

$$\mu \to e + \bar{\nu} + \nu'$$
$$\mu \to e + \bar{\nu} + \nu$$
$$\mu \to e + \bar{\nu}' + \nu'$$
$$\mu \to e + \bar{\nu}' + \nu$$

all quite indistinguishable, experimentally. Since the rate of muon decay is measured, we must require the following condition on the four coupling constants,

$$|g_1|^2 + |g_2|^2 + |g_3|^2 + |g_4|^2 = G^2 . \qquad (5)$$

Furthermore, if all these interactions are mediated by vector bosons, we must be careful that the decay mode $\mu \to e + \gamma$ is not induced. Clearly, only g_2 and g_3, corresponding to unit violation of muon number, are involved. Cancellation of this unwanted decay mode requires

$$g_2 + g_3 = 0 . \qquad (6)$$

Consider the experiments that can help determine these parameters. Imagine the processes

$$\nu' + e \to \mu + [\nu] \qquad (7a)$$
$$\bar{\nu}' \to e + \bar{\mu} + [\bar{\nu}] \qquad (7b)$$
$$\nu' \to \bar{e} + \mu + [\nu] . \qquad (7c)$$

The square bracket indicates that the identity of the final neutrino or antineutrino is not measured ; one observes only the incoherent sum of the cross sections for either kind of neutrino. The latter two processes require the intervention of a Coulomb photon, and take place in the neighbourhood of a heavy nucleus. Note that the cross sections for reactions (7) depend on the combination of coupling constants

$$|g_1|^2 + |g_2|^2 .$$

Another combination of coupling constants, $|g_2|^2 + |g_4|^2$ is determined from a study of the conventionally forbidden reactions

$$\bar{\nu}' + e \to \mu + [\bar{\nu}] \qquad (8a)$$
$$\bar{\nu}' \to \mu + \bar{e} + [\bar{\nu}] \qquad (8b)$$
$$\nu' \to \bar{\mu} + e + [\nu] \qquad (8c)$$

To obtain further information about the g_i one must perform experiments with electron neutrinos. This is a difficult but not impossible task, since almost 10 % of the neutrinos produced in kaon decay are electron neutrinos.

From the processes

$$\bar{\nu} + e \to \mu + [\bar{\nu}] \qquad (9a)$$
$$\nu \to \bar{e} + \mu + [\nu] \qquad (9b)$$
$$\bar{\nu} \to e + \bar{\mu} + [\bar{\nu}] \qquad (9c)$$

one may, in principle, measure $|g_1|^2 + |g_3|^2$; and from the processes :

$$\nu + e \to \mu + [\nu] \qquad (10a)$$
$$\nu \to \bar{e} + \mu + [\nu] \qquad (10b)$$
$$\nu \to e + \bar{\mu} + [\bar{\nu}] \qquad (10c)$$

one may determine $|g_3|^2 + |g_4|^2$.

In view of (5), we see that there are just three independent coupling strengths, but experiments can determine only two of them. Moreover, if we assume (6), there remain two undetermined parameters, of which experiment can determine only one, i. e.

$$| g_1 |^2 + | g_2 |^2 = | g_1 |^2 + | g_3 |^2 = \cos^2 \beta G^2$$
$$| g_3 |^2 + | g_4 |^2 = | g_2 |^2 + | g_4 |^2 = \sin^2 \beta G^2 \ . \quad (11)$$

The parameter $\cos^2 \beta$ can be measured, but no experiment can distinguish between a violation of muon number by one ($g_2 = - g_3 \neq 0$) or by two ($g_4 \neq 0$). In any case, the measurement of this parameter is the crucial test of the validity of the conservation law for muon number.

Finally, we come to the diagonal leptonic interactions, for which we consider the following phenomenological Lagrangian

$$\begin{aligned} & f_1 \{ \bar{e}\gamma^\lambda(1 + \gamma_5) \nu \} \{ \bar{\nu}\gamma_\lambda(1 + \gamma_5) e \} + \\ & + f_3 \{ \bar{e}\gamma^\lambda(1 + \gamma_5) \nu' \} \{ \bar{\nu}' \gamma_\lambda(1 + \gamma_5) e \} \\ & + f_2[\{ \bar{e}\gamma^\lambda(1 + \gamma_5) \nu \} \{ \bar{\nu}' \gamma_\lambda(1 + \gamma_5) e \} + h.\ a.] \\ & + h_1 \{ \bar{\mu}\gamma^\lambda(1 + \gamma_5) \nu' \} \{ \bar{\nu}' \gamma_\lambda(1 + \gamma_5) \mu \} \\ & + h_3 \{ \bar{\mu}\gamma^\lambda(1 + \gamma_5) \nu \} \{ \bar{\nu}\gamma_\lambda(1 + \gamma_5) \mu \} \\ & + h_2[\{ \bar{\mu}\gamma^\lambda(1 + \gamma_5) \nu' \} \{ \bar{\nu}\gamma_\lambda(1 + \gamma_5) \mu \} + h.\ a.] \, . \end{aligned}$$

The conventional theory is obtained by the choice $f_1 = h_1 = G$ and $f_2 = f_3 = h_2 = h_3 = 0$. The terms involving f_2 and h_2 violate muon number (and electron number) by unity ; the remaining terms conserve muon number.

Once again, there are more parameters than may be determined by feasible experiments. Only the combinations $| f_1 |^2 + | f_2 |^2$, $| h_1 |^2 + | h_2 |^2$, $| h_2 |^2 + | h_3 |^2$ and $| f_2 |^2 + | f_3 |^2$ are presently measurable. In particular, experiment cannot distinguish $f_2 \neq 0$ from $f_3 \neq 0$. Hence, a violation of muon number cannot be proved by a study of the diagonal interactions. However, it is precisely on the question of diagonal interactions that several recent models differ. We quote in Table II predicted values of f_i in these models. (All models I know of predict $f_i = h_i$, for they incorporate μe universality).

TABLE II

	Usual theory	*KLGG*	*GIL*
f_1	G	pG	$(q + \frac{1}{2})^2 G$
f_2	0	0	0
f_3	0	0	$(q - \frac{1}{2})^2 G$

Here, KLGG indicates the model of Gell-Mann, Goldberger, Low and Kroll [2] wherein the diagonal interactions are renormalized differently from non-diagonal interactions, and p is an unknown positive number. GIL [3] stands for the model of Iliopoulos, Maiani and myself, and q is again an unknown positive number : in this model elastic scattering of ν' from e is expected. Soon, one hopes, experiment will exclude one or more of these possible models.

References

[1] Another extreme possibility, now ruled out by experiment, is the « neutrino-flip hypothesis » : $\nu_1 = \nu_4$, $\nu_2 = \nu_3$, $< \nu_1 | \nu_3 > = 0$.
[See FEINBERG (G.), GURSEY (F.) and PAIS (A.), *Phys. Rev. Letters*, 1961, **7**, 208.
BLUDMAN (S.), *Phys. Rev.*, 1961, **124**, 947.]
[2] GELL-MANN (M.) et al., *Phys. Rev.*, 1969, **179**, 1518.
[3] GLASHOW (S.) et al., *Phys. Rev.*, 1970, **D 2**, 1285.