Shedding light on diverse cultures of mathematical practices in South Asia: Early Sanskrit mathematical texts in conversation with modern elementary Tamil mathematical curricula

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SCIENCE IN THE FOREST, SCIENCE IN THE PAST

Shedding light on diverse cultures of mathematical practices in South Asia: Early Sanskrit mathematical texts in conversation with modern elementary Tamil mathematical curricula

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Discourses promoting alternative sciences—that is, traditional "scientific" systems as opposed to modern "Western" science—are part of a wider reflection in South Asia on how to shape the future. History of science, then, is a very politicized affair. We will describe how two contrasting practices of number—measures and computations—can be documented in South India as they appear in early Sanskrit mathematical treatises and commentaries (seventh–twelfth centuries) and also in elementary mathematical curricula in Tamil (seventeenth–twentieth centuries). We will question whether they should be described as two different mathematical ontologies.

Keywords: India, computation, measuring, approximations, areas, gold, politicized history of science

The politics of alternate modernities

Since India's Hindu-nationalist Prime Minister Narendra Modi has been in office, he and other government officials have made headlines with surprising statements concerning the scientific feats of past Hindus: for instance, he proclaimed that ancient Hindus knew how to perform cosmetic surgery, as proved by the existence of Gaṇeśa, the god who has the shape of a human with an elephant head;[1] the Chief Minister of the state of Tripura has recently claimed that at the time of the *Mahābhārata*, Hindus had internet and satellites, since Saṃjaya Galvani could give vivid accounts and updates of the formidable Kurukṣetra battle to King Dhṛtarāṣṭra;[2] and one could come up with other such examples. These laughable claims of Indian officials can be understood as a sign of contemporary India's enchantment and disenchantment with science (Raina 1997). It reveals the complex social, political, but also emotional relation that South Asia has to its past and present knowledge and know-hows. Indeed, part of South Asia's postcolonial identity-building has to do with an engagement with its "science of the past," but also its "science of the forest" (here, forest being understood more largely as "rural India"). The most famous example is the gandhian reclamation of economic autonomy by promoting traditional know-hows, such as *khadi* (hand-spun, hand-woven cloth). Such an engagement carries many diverse political nuances and takes many forms. Many paths, models leading to what could be a new modernity (Prakash 1999) are thus explored. Imagined alternatives of what could be a non-Westernized, non-Christian, non-capitalist, or non-imperialist etc. appropriation of science and its power are investigated. Villages and forests, then, can be seen as potentially sowing the seeds of the future society as activists turn to them to learn how to harvest water, find medicinal plants, or teach science

1. https://www.theguardian.com/world/2014/oct/28/indian-prime-minister-genetic-science-existed-ancient-times.
2. https://economictimes.indiatimes.com/news/politics-and-nation/internet-existed-in-the-days-of-mahabharata-tripura-cm-biplab-deb/articleshow/63803490.cms; https://www.bbc.com/news/world-asia-india-43806078.

classes.[3] As action for the future, the recovering of contemporary local know-how and knowledge also nourishes approaches to history; notably, in the case we will explore here, the history of mathematics.[4]

In 2005 I embarked, with Senthil Babu of the French Institute of Pondicherry, on an ethno-mathematics (lit. anthropology of mathematics) project; we visited some villages of the Nagapattinam area in South Eastern India. We were struck at the time by the ways in which numeracy and the ability to make conversions were at the heart of the negotiations between landless, low-caste laborers, and the higher-caste owners. Converting hours of labor, in relation to areas of sowed fields, yielding given capacities of more or less refined grain, would be what would determine the amount that such laborers should be paid. In such transactions, tensions relating to evaluations of fractional parts, of measures falling between standards of measurement, were obvious. Much of the payment was in-kind. Part of the village economy rested on value rates enabling exchanges in-kind with the help of capacity vessels: this amount of rice can pay for that amount of oil or clarified butter. In this system, the capacity measures were local, as were the laborers' measures of areas. The landowner and, crucially, the surveyor knew how to navigate between the local measuring unit system and the metrical system that is officially in use all over India today. Here, then, what numbers you know how to compute with, with what units you know how to make conversions, how much you can convert kind into money, were markers of the power you had or did not have, but also of a knowledge that, if acquired, could empower you. Further, the economy and

3. For Babu (2015) this would rather be about different imagined publics for which specific mathematics are shaped.

4. The reappropriation and reactualization of one's scientific history, questioning how to situate it in today's world, are something that has been at work, of course, for a very long time. Studies now document how before India's independence itself, at the end of the nineteenth and beginning of the twentieth century, all sorts of hermeneutical schemes and hybridization were at work, from the adaptive translation of De Morgan's mathematical manuals into Marathi (Raina 2016) to the reading of the Vedas as providing the fundamental elements of chemistry (Dodson 2005), all of which, of course, are to be understood in their specific contexts.

system of value for the laborer seemed a parallel system to the more mainstream one of the national economy. There could be many economies of values, and as long as they were easily convertible into one another, they could yield power. The questions we had and could not answer were these: Is the laborer's value system entirely convertible into our terms? Should it be?

Both Babu and I were historians of mathematics, improvising as ethnographers. And this experience became a lesson in history writing: it first revealed precisely how mathematical knowledge and practices could be a point of conflict. It also raised the question of how, and with what sources, we were uncovering such a history: Were we writing the history of the winners, of the power holders? We also couldn't help but raise the question—which Marilyn Strathern justly criticizes in her essay—in terms of traditions. What from the past explained the present in this case?

In what follows we will examine what can be said, in the past, of what appears as two separate cultures of computations and measuring: medieval (seventh–twelfth century) Sanskrit mathematical treatises and commentaries and modern (seventeenth–twentieth century) Tamil school primers, before coming back, in the conclusion, to the question of the existence of different mathematical ontologies.

Most of the history of mathematics of South Asia has been written using scholarly documents in Sanskrit. Sanskrit, an Indo-European language, was at first a high-caste Hindu, brahmanical language. By the fifth century CE, the language was used by a certain cosmopolitan elite, which might live outside of South Asia, might not be a Hindu, and not necessarily be a brahmin. Not much is known of the institutions of learning in which such texts were produced, studied, or copied. Astral science is believed to have been both a scholarly field of study and a family-related occupation. Training would have been attended to either within one's family or through prestigious schools (Plofker 2009: 178–81). For the most part, however, the contextual information—including what we can glean about teaching—is embedded in the texts themselves.[5] In all cases, Sanskrit mathematical texts represent high-brow mathematics. Two different types of texts have been transmitted to us. The first, the most prestigious, is in the form of chapters of theoretical astronomical texts. These were mostly transmitted by continuous copying. Another kind of text, much less copied, often referred to as "mathematics for worldly practices" (*loka-vyavahāra*), is known to us by the chance find of one or two manuscripts. These sources document different mathematical practices, but they also share many common technical tools and topics. One of the characteristics, and important values of what Sheldon Pollock (2006) labels the "Sanskrit Knowledge System," is its aim at being universal. Sanskrit treatises and their commentaries do not want their scholarly production to rest on circumstances of time and place, which are regularly erased. Until roughly the seventeenth and eighteenth century, the mathematical texts we have articulate and think of themselves as an immemorial discipline (*gaṇita*): a new text is always the reframing of a preceding treatise or of an orally transmitted doctrine (*śāstra*). This can partly explain why histories of mathematics in Sanskrit have provided a very homogeneous and ahistorical point of view on this literature.[6] We know that astral and mathematical texts written in Sanskrit in the fifth, seventh, or twelfth century were still in circulation in different parts of South Asia during the eighteenth and nineteenth century. However, this Sanskrit literature lived side by side with texts in other Indian languages, generically called "vernaculars." Sources documenting past mathematical practices in Telugu, Bengali, Tamil, Marathi, Malayalam, Persian, and in various forms of Prakṛt exist, ranging mostly from the seventeenth century onward, except for the Jain Prakṛt texts that

5. The problem, then, is to find ways of retrieving such information from them. Ganeri (2008) and Keller (2015) use Speech Act Theory to do so.
6. Of course, other reasons can be added to this one, such as the importance of constructing a national scientific heritage, or the belief that when dealing with exact sciences historicity is not what is at stake but rather the technical contents of a text.

were canonized in the sixth–seventh century of our era. The texts have been edited and sometimes studied in the last fifty years, but the mathematics they testify to has remained in the margins of the writings on the history of mathematics in South Asia. Circulation of mathematical and astral knowledge between the vernacular and the Sanskrit is documented essentially as transmissions from the Sanskrit to the vernacular (Sarma 2011). It is possible, however, to imagine that the reverse could at times be true as well; notably, through the circulation of mathematical problems and riddles. We will discuss these transmissions below. Babu (2015) documents the elementary schooling in Tamil in the early nineteenth century in mathematics and shows that brahmanical and rich children were not schooled in the same way as other children. This is discussed in further detail in the next section. India has long been perceived as the champion of arithmetics, the continent from which zero and our way of computing with decimal place value notations have come. Do vernacular documents show alternative systems of noting numbers and dealing with values and measuring units? Do they contrast with what can be found in Sanskrit texts? Do these practices reflect distinct social contexts? And are these systems mutually exclusive: Does adopting one mean you can't convert back to the other?

Computing or not with the decimal place value notation

In what follows, I present what may have been two very different cultures of quantification and culture that can be documented in South Asia—more specifically, in the Tamil world of the seventeenth century and until the late nineteenth century. The decimal place value notation was a technical and scholarly device found in Sanskrit lore from at least the fifth century CE all over the Indian subcontinent. In addition, another mode of noting numbers existed: the "Tamil numerals," a decimal notational nonpositional system, including special signs for small fractions, which, Babu suggests, was widespread among "practitioners" (accountants, surveyors, etc.).[7] To execute operations on values noted with Tamil numerals, the tables that were part of the elementary Tamil curriculum could be used: operations such as multiplications and division were not "executed" at least; their resolution was made through the more or less elaborate use of the entries of tables. Second, in the Sanskrit corpus, the decimal place value notation figures as a tool for computing on numbers, without paying attention to the unit in which its value makes sense. Yet, by contrast, in the widespread Tamil primer the *Kaṇakkatikāram* (circa fifteenth–eighteenth century), computations were always carried out in such a way that they were meaningful in terms of measuring units each step of the way.

These are very preliminary results, which rest on what is still a very fragmentary access to proper documentation. For instance, the Sanskrit mathematical texts in circulation in Tamil Nadu in the seventeenth–nineteenth centuries need to be adequately documented, to specify more precisely the kind of practices that would have been familiar at that time and place and in real conversation with the Tamil texts. Their practices of value, number, and quantification will be presented while dealing with two common problems: the computing of areas and the purity of melted gold.[8]

Computing approximate areas of quadrilaterals: With or without specific measuring units?

In the mathematical chapter of the *Brahmasphuṭasiddhānta*, a theoretical astronomical text authored by Brahmagupta in 628 CE, the first part of a versified rule in Sanskrit provides the

7. The existence of this numerical system is well attested. Other numerical forms as well, including very ancient systems of numerical noting, which might predate the decimal place value notation.]
8. A new interest in manuals in vernacular languages should lead to a series of new publications. Here I use the very fragmentary elements that I have gleaned from Babu (2015). Hopefully, this thesis will be followed by more substantial and extensive reeditions and translations. Roy Wagner as the new chair of History and Philosophy of Mathematical Sciences at the ETZH (Swiss Federal Institute of Technology) in Zurich is also translating similar material in Mayalayam. Both have launched on a new program to create a census, edit, and study the vernacular sources of South India.

rough area of trilaterals and quadrilaterals): “BSS 12 21ab. The gross area is the product of half the sum of the sides and counter-sides of a tri (or) quadrilateral” (Dvivedin 1902).

In his commentary on this chapter, Pṛthūdhaka provides some examples of computations of such rough areas. The computations all deal with numbers with no associated measuring unit. The problems, although dealing with *kṣetra*s (lit. “field”), should be understood in this technical context as concerned with “geometrical figures.” Since the chapter contains also a rule defining the decimal place value notation, and rules to compute multiplications, cubes, and cube roots with this notation, we can assume that the simple and more complex computations with fractions all used the decimal place value in this case. This is indeed how it can be found in the late manuscripts we have of this text. This rule was subsequently found throughout Sanskrit texts devoted to mathematics, outside of the realm of astronomy.[9] The measuring units used in Sanskrit mathematical and astronomical texts were standard, and could also be found in other texts of Sanskrit literature; notably, in treatises of state administration.[10] According to Sreeramula Rajeswara Sarma, they were called the “Magadha Units” (*māgadhamāna*) and were maybe part of the reverence shown by the discipline to its past.[11] Were these measuring units just theoretical ones, or were they those standardly used in South Asia, as part of the means for the educated to convert and compute, whatever their local circumstances? In the texts to be examined subsequently, the lengths are given in the famous linear measure, *hasta* (forearm, sometimes translated cubit), which measures twenty-four linear *aṅgulas* (breadth of a finger). The square *hastas* in the following

9. As noted in Shukla (1959: 87 n. 2), in the *Pāṭīgaṇita* of Śrīdhāra (eighth–tenth century) (in verse 111cd); in the *Gaṇitasārasaṃgrāha* of Māhavīra, (vii, 7ab), in the *Mahāsiddhānta* of Āryabhaṭa (xv, 66) or the *Gaṇitakaumudi* of Narayaṇa (II, verse 8), etc.
10. Those used in what follows are presented in an Appendix at the end of this essay.
11. Sarma: “The Sanskrit texts on arithmetic employ in their sums the so-called *Māgadhamāna*, i.e., units of measurement, weight, and coinage, which are said to have been prevalent in Magadha in ancient times (probably when Āryabhaṭa was writing at Kusumapura) and not the contemporary units” (2011: 204). This remark can be nuanced, as close study shows that units were far from being as homogeneous as they seem.

problems are made of twenty-four square *aṅgulas*: the ratio is the same as in the linear case. Thus, square hastas are not made of linear *hastas*, as explained in the Appendix on measures at the end of the essay.

In the anonymous and undated commentary on the *Pāṭīgaṇita* of Śrīdhāra, problems are formulated in words with measured values. They are solved by translating these problems into numerals with no associated measure, on which computations are carried out. The result of the computation, can then be restated, with an associated measuring unit and a certain shape (fractions are not generally found alone). In what follows, the computation of the area of a rectangle is carried out: "PG.115.Ex.123. Tell me the computation when a rectangle's base and mouth amount to five and a half *hastas*, while the flanks and middle height are three *hastas*" (Shukla 1959; Sanskrit p. 161–62).

The rectangle's sides in this example are named as conventionally as those of any quadrilateral, with an implicit orientation: the base (*bhū*) is usually horizontal, the face (*vadana*) is its opposite side, there are both flanks-sides (*pārśva-bhuja*), and a height (*lamba*), which extends from the middle of the base and the face, since it is a rectangle. [12]

The edition of the text provides a diagram, which notes the measures of each length, as shown in Figure 1.

12. The resolution of the problem also extends to the case were only the value of the height is considered.

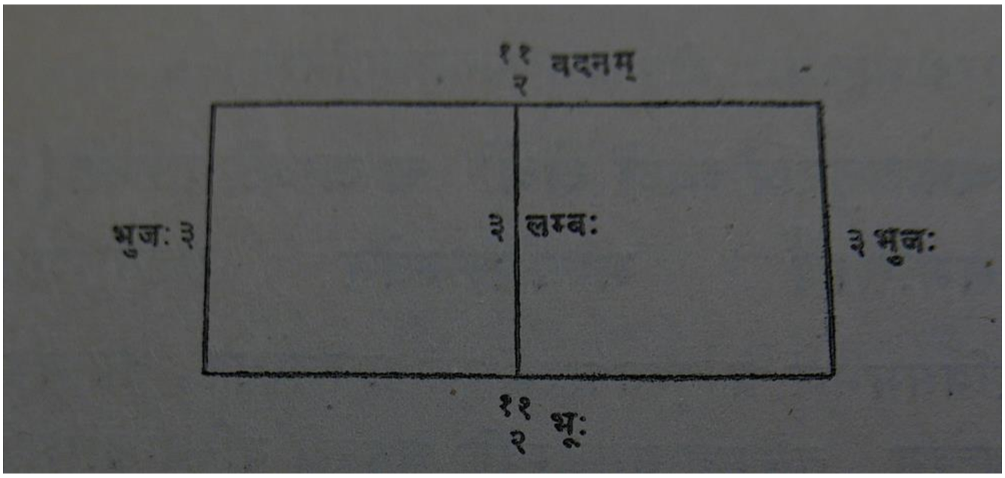

Figure 1: The rectangular diagram in the problem of the *Pāṭīgaṇita*, as in Shukla (1959).

The treatise's rule states that "the product of half the sum of the sides and counter-sides" should be taken. The "base" and its opposite side, "the face," is according to the statement of the problem, five and a half *hastas*, which is noted 5.

1

2

The two other opposite sides measure 3 *hastas*.

The computation, then, following the rule stated by Brahmagupta and restated by Śrīdhāra, first computes the sum of each couple of sides, (5 + ½) + (5 + ½) = 11, 3 + 3 = 6, and then their halves, 1½, 3, and then the product of both: 33/2. This is then converted, and since 33/2 = 16 + ½, and since 1 square *hasta* is 24 square *aṅgula*, the result obtained is "sixteen [square] *hastas* twelve [square] *aṅgulas*."

The resolution literally reads as follows:[13]

Procedure: the base is 5 the mouth 5

1 1

13. () indicates the editorial additions made by K. S. Shukla. (. . .) indicate that part of the text has been skipped.

2 2

(. . .)

(The sum of both sides and opposite sides 11|6), their half $\frac{11}{2}$| 3, their product $\frac{33}{2}$

from which the result is just that: sixteen *hastas*, twelve *aṅgulas*. (Shukla 1959; Sanskrit p. 132)

The numbers in the diagram, like those of the computation that follows, are noted, for their integral part, with the decimal place value notation. No mention is made of their units. The result of the computation is stated twice, once in a numerical form, then restated and converted specifying its measuring units in words. It is also striking that in the diagram the number associated with the “base” and the “face” is not 5 + ½ but 11/2. Indeed, the value stated in the text of the problem is first converted into numbers noted on a working surface. Here, the resolution does not seem to require the transformation of the fractional form of an integer increased by a fraction smaller than one (5 + ½), into a simple fraction with numerator and denominator (11/2), but the edited diagram does.[14]

It is not clear why such elementary computations are detailed here, amid much more complex problems, but they do highlight an articulation between values that are stated in problems and answers, on the one hand, and numbers noted in specific manners to compute with, on the other hand. Further, the two texts taken as examples can easily be seen as exhibiting some of the spectrum within the Sanskrit corpus itself: on the one hand, a prestigious theoretical astronomical text of the seventh century, and on the other hand, an undated (but seemingly quite late) commentary on a tenth century text concentrating on

14. This might be the choice of the editor, Shukla, or that of a scribe. The location of this unique manuscript of this commentary is not known today, so it is difficult to check.

mathematics as a topic of worldly affairs. However, they both share a way of emphasizing the computations on numbers over measured values.

As mentioned above, a corpus of mathematical texts in Tamil, a Dravidian language of South East India, has been edited and studied in the last fifty years. This corpus encompasses texts of different kinds and times.[15] In what follows, a mathematical text written in classical versified Tamil, which might have been a canonical primer if not an actual textual genre, the *Kaṇakkatikāram* (hereafter abbreviated as *KK*)[16] will be used as a source. A first author of such a text, Kāri Nāyaṉār, would have lived in the fifteenth century in what is today the Nagapattinam district of Tamil Nadu (Babu 2015: 32), while manuscripts from the eighteenth century onward can be found throughout Tamil Nadu, suggesting a widespread textbook. The edited text, which attempts to collect in one text all the variations of thirty-three different manuscripts, presents itself as an "exposition of mathematics to the world," and is composed of rules in classical Tamil verses and problems, some versified, others in prose, and some cursory resolutions. Among the standard topics it treats, problems of land measuring and alloys will be singled out here. The mathematical practices documented in this text will be associated with the two texts that formed in the early nineteenth century (and probably before that) the basis of education/learning, in rural areas, for non–high caste children: the lexical lists, called *Poṉṉilakaṃ* and the multiplication tables, called *Eṇcuvaṭi*. Babu (2015: 127) has shown how they were important parts of the village schools (also known as veranda or *Tiṇṇai* schools) that English observers sometimes called the "multiplication schools." Babu (2015: 83–103) further argues that the *KK* should be

15. For a preliminary study of computations in Tamil inscriptions, see Subbarayalu (2012) and Selvakumar (2016).
16. Three editions of the texts are noted by Babu: Satyabama (1998), Subramaniam (2007), Satyabama (2007) For the status of the text, see Babu (2015: 32–33).

understood as a textbook to train practitioners such as the surveyors (*veṭṭiyāṉ*), the tax collectors, or accountants (*kaṇakkaṉ*).

Tamil numerical notations extended to higher numbers and to fractions smaller than one, in many forms. And, as demonstrated by Babu (2015: 119ff.), the teaching associated learning by heart, learning how to write the notations, and learning their values. The *Eṇcuvaṭi* tables, as edited by Babu, are hybrid in terms of numbers and measured values. Some state the measuring unit of the numbers considered. Further, the tables incorporate examples of how to find the value of a product when such a multiplication is not directly tabulated. For example, as shown in Figure 2, to multiply 17 x 17 *kuḻi* (an area measuring unit described below as well as in the appendix at the end of this essay), the value is found by considering the entries for 10 x 10, 10 x 7 (twice), and 7 x 7, and by adding them progressively.

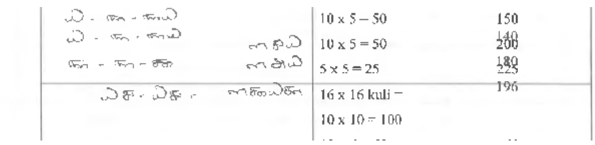

Figure 2: The entry for 17 x 17 *kuḻi* in the Eṇcuvaṭi, edited in Babu (2015).

In the *Kaṇakkatikāram* we can see how an approximation of quadrilateral areas was computed:

> KK. 81 Add one side to another and halve and multiply the resulting side by the larger one. If you do so, you will obtain all the areas (*nilaṅkaḷ*) of the world like the rays of the sun spreading on earth. (Babu 2015: 46, quoting Subramaniam 2007: 125)

The general rule thus approximates the surface of any quadrilateral by approximating it to the area of a rectangle whose sides would be half the sum of the quadrilateral’s opposite sides. As in Sanskrit texts, orientations of figures are provided by cardinal directions.

In South India, before and during the Chōḻa rule (850–1250) the *kuḻi* was an important area measuring unit. The *kuḻi* corresponds to a square having for side the length of a measuring rod (in linear measures of *kōl*). The *kuḻi* served as a surface unit by which other surface units could be determined (Subbarayalu 2012: 83; Babu 2015: 135). The rule stated above is followed by a problem and its resolution:

> right/South *kōl* 13 left/North *kōl* 11 therefore *kōl* 24, half of that is 12; West/lower *kōl* 9 East/upper *kōl* 19, therefore *kōl* 28, its half is 14; multiply this 10 x10 = 100 , 4 x 10 = 40 , 2 x 10 = 20, 2 x 4 = 8 . Therefore <the result is> 168 *kuḻi*; this is how you say it. (Babu 2015: 46, quoting Subramaniam 2007: 125)

In this problem a quadrilateral has four sides measured in *kōl*, whose values are given as couples of opposite sides. The "flanks" of the quadrilateral in the Sanskrit text are here its South and North sides, respectively 13 and 11 *kōl*. The opposite "base" and "faces" are here the West and East sides, 9 and 19 *kōl*.

The rule provided above involves first: "Add one side to another and halve." Thus, for the first pair of sides, the computation is carried out, 13 + 11 = 24 *kōl*, its half is 12. For the second pair, 9 + 19 = 28 *kōl*, its half is 14. The next step of the process is then: "multiply the resulting side by the larger one." Although it is unclear what this means in terms of assessments of the respective "mean sizes" just computed, we understand that the product of 12 by 14 has to be carried out. Here, as in the multiplication table examined above, the product of (10 + 2) x (10 + 4) is carried out, the individual products are made explicit and their sum 100 + 40 + 20 + 8 = 168 computed. This last computation provides the result.

In this resolution, the execution is such that at each step the measuring unit of what is being computed is always known. And this is often (although not always) stated. When the area is computed, however, and the product of 12 by 14 is carried out, the detailing of how values are taken from a table seems to be noted, as in multiplication tables. Here, then, when using multiplication tables, the computation is carried out on numbers, with which measuring units are not associated. But as soon as the result is input into the resolution of the problem it exists as a measured value again, and the result is too.

Another problem deals with proportions of areas and their different uses:

> If the total *vēli* is 1000, and if 1/10th has been sown; 1/8th has been transplanted; 1/4th has nurseries and 1/2 has matured paddy, how much was the wasteland and how much was the cultivated area? (Babu 2015: 49, quoting and translating Subramaniam 2007: 160)

The solution of this problem involves computing 1000 x 1/10, 1000 x 1/8, 1000 x 1/4, 1000 x 1/2, and adding all the results. At each step of such a computation the results would always be in *vēli* (another area measuring unit). It is possible to arrive at a resolution using the tables, the lexical lists (*Poṉṉilakaṃ*), and the multiplication tables (*Eṇcuvaṭi*) to retrieve the value of each product.[17]

These examples illustrate how tables of multiplication could have been used to execute more complex products, without the decimal place value notation. We can thus contrast the following: Sanskrit sources (the *Brāhmasphuṭasiddhānta* and the *Pāṭigaṇita*, for example), which compute using the decimal place value notation and whose emphasis is on the different operations executed on numbers not associated with measuring units when computing areas, with the *Kaṇakkatikāram* in which the area is computed with an attention given to the measuring units linear and square at every step. Numbers—for example, values without a measuring unit—would come into play in the intermediate steps of retrieving values in multiplication tables when computing products.

17. 1/10th of a 1,000 can be obtained by reading the multiplication table from right to left of *Encuvatti Mēlavai ilakkam*, and retrieve 100 (Babu 2015: appendix III, 52). 1/8th is a standard fraction, having its own sign in the "Tamil numeral system"; 1000 x 1/8 = 125 is the last line of the part devoted to 1/8th in *Mēlavai ciṟṟakkam* (Babu 2015: appendix III, 62). 1/4th is also a standard fraction; it also has its own multiplication table. 1000 x ¼ = 250 is the last line of the part devoted to 1/4th in *Mēlavai ciṟṟakkam* (Babu 2015: appendix III, 67). 1000 x ½ = 50 is the last line of the part devoted to 1/2 in *Mēlavai ciṟṟakkam* (Babu 2015: appendix III, 68). How additions (100 + 125 + 250 + 500 = 975) and subtractions (1000 – 975 = 25) were computed is not made explicit here.

“Gold computations”

Similar contrasts of practices with numbers, measured values, and computations can be documented in problems about the fineness of gold, its price, or the mixing of silver and gold. It is striking that in the Sanskrit texts dealing with this quite standard topic (*suvarṇa-gaṇita*) (Sarma 1983), computations are carried out mechanically with numbers. For instance, in the canonical arithmetical *Līlāvatī* by Bhāskara (b. 1104), a problem of the cost of gold is solved by an inverse Rule of Three (*vyasta-trairāśika*). After the quantities are displayed on a grid on a working surface, represented by a “setting” (*nyāsa*) clause in the text, a multiplication is carried out on the left and a division is made on the right. The answer here is given in noted numbers, with no measuring unit associated with them. The *varṇa* (lit. color) of gold stands for its purity, a measure not unlike the carat, the *gadyāṇaka* is a measure of weight, and the *niṣka* a gold coin (Āpaṭe 1937: 75; Colebrooke 1817: 35, 46–47).

> An example concerning the price of gold according [to its] *varṇa*:
>
> If a ten *varṇa* gold [weighing] a *gadyāṇaka* is bought for a *niṣka* say then how much [can be bought, with the same amount of money of a] fifteen *varṇa* gold?
>
> Setting
>
> 10 | 1| 15|
>
> the result is 2
>
> 3

Here then, 1 x 10 (multiplication on the left) was divided (on the right) by 15, and a fraction obtained, 2/3. No measuring unit is stated for the result, which is implicitly in *gadyāṇaka*s.

In the portion of the same treatise devoted to “gold computations” (*suvarṇa-gaṇita*), a similar phenomenon can be found: a tabular display enables the mechanical carrying out of a problem. First a rule is stated, to solve a problem where several parcels of gold of different weights w_i and purity v_i are melted together. The weight of the melted result is assumed to be the sum of the previous weights (Σw_i). The purity of the new lump of gold, V, is computed with a procedure that amounts to $V = \Sigma w_i v_i \backslash \Sigma w_i$.[18] The procedure then amounts to computing the sum of the products $w_i v_i$, then the sum of w_i, and divide the first by the second.

The rule is followed by a solved example (Āpaṭe 1937: 99, verse 104–5; Colebrooke 1817: 35, 46–47):

> An example:
>
> Parcels of gold weighing severally ten, four, two, and four *māṣās*, and of *varṇa* thirteen, twelve, eleven, and ten respectively, being melted together, tell me quickly, merchant, who art conversant with the computation of gold, what is the *varṇa*s of the mass? (…)
>
> Setting: 13 12 11 10
>
> 10 4 2 4
>
> The measure of the *varṇa* of gold obtained when melted, 12.

Here, the setting shows how the computation can be done mechanically. The *varṇa*s (v_i) are on the top row, the weights (w_i) of each parcel on the lowest row. To compute the weight of the whole melted gold, the sum of the cells of the lower row is made (20). The product of the numbers in each column can be made (13 x 10 = 130, 12 x 4 = 48, 11 x 2 = 22, 10 x 4 = 40) and the sum of the products computed (130 + 48 + 22 + 40 = 240), to be able then to divide the latter by the first (240/20) and obtain the result (12). The products and their sum are typically numbers to which no specific measuring unit can be associated.

18 The sum of the products of the *varṇa* and [weight of several parcels] of gold being divided by the [weight of the melted] gold, the value of the *varṇa* of the [melted] gold [is obtained]. The *varṇa* is [obtained] when [the previous result is] divided by the [weight of] the purified gold, the amount of purified gold when divided by the *varṇa* (Āpaṭe 1937: 99, verse 103; Colebrooke 1817: 35; 46–47, verse 102–3). Colebrooke's verse numbering has been adopted here and in what follows. Note that gold and weight of gold are one and same word here.

Similar but different "gold computations" can also be found in the Tamil *Kaṇakkatikāram*. In this context, the *māttu* is a gold weight as well as a way of measuring the purity of gold, the *kalañcu* and *paṇavetai* measures of weight, and the *paṇam* a coin.[19] In the problem given in this treatise here, and the rule that follows, the computation that involves first a division and then a multiplication ensures that at each step the measuring unit of what has been computed is known:

> If the cost of one *kalañcu* of gold with ten *māttu* is fourteen *paṇam*, what would be the cost of an eight *māttu* gold? Then you divide the *paṇam* by the *māttu* of the given gold and multiply it by the other given *māttu*. (Babu 2015: 51, quoting Subramaniam 2007)

The problem is seen here as a kind of Rule of Three since the weight of gold is stable. The first division gives you the price of gold per *māttu*, and the second the price sought.[20]

It is well established that Sanskrit sources of diverse times and contexts dealing with areas or gold computations (but the topics could be largely extended) all use the decimal place value notation to compute. These computations are carried out with numbers whose value and relation to measuring units can be made explicit when stating a problem or providing the final result, but not while the different steps of the computation are executed. In the *Kaṇakkatikāram*, which we have associated with the lexical and multiplication tables that were taught to children in Tamil Nadu from the eighteenth century, some problems seem to be solved in such a way that each step of the computation accounts for the units of the values considered. Only when products are carried out, and multiple entries of a multiplication table

19. For weight standards as documented in Tamil texts and epigraphy see Babu (2015: 7–8); for gold computations, see Babu (2015: 50); Sarma (1983). For the *paṇa* (tamil *paṇam*), see the appendix at the end of this essay.
20. 14/10 = 1 + 2/5 *paṇam*s, which multiplied by 8 would have been, 9 + 3/5 *paṇa*s. We do not know if such a result accounts for *paṇam* denominations. Divisions may have beeen carried out by inversing tables of multiplications.

need to be retrieved and added, would numbers be considered with no associated measuring unit.

We have thus seen here a testimony of what might have been two different mathematical cultures belonging to different social contexts: the scholarly world of Sanskrit mathematical lore opposed to elementary school texts in Tamil.[21]

Is it Tamil *versus* Sanskrit?

Should we see these two cultures as exclusive of one another and characterized by the language they use? Things are not so simple, of course. Thus, in this example from the *Kaṇakkatikāram*, the fineness *māttu* of an alloy is measured against its weight in standard *paṇaveṭai*:

> When 4 *paṇaveṭai* of gold with 7 1/2 *māttu* are mixed with one *paṇaveṭai* of silver, what will be a *māttu* of the resulting gold? The original *māttu* has to be multiplied by its weight to be divided by the total weight, to yield the final refinement *māttu*. 7 1/2 x 4 *paṇaveṭai* = 30; weight of gold 4 and silver 1- (their sum is) 5; 5 is the total weight in *paṇaveṭai*; the new *māttu* is 30/5 = 6. (Babu 2015: 51, quoting Subramaniam 1999: 224–26)

Here, then, in contrast to the result seen in the previous section, but using a procedure that resembles the one we have seen in the *Līlāvatī*, the procedure to find the fineness of a gold and silver alloy requires computing:

- the product of fineness and weight of the gold $w_i v_i$;
- the sum of the weights w_i of the gold and silver;
- the division of the product by the weight: $w_i v_i / w_i$;

which would provide the fineness of the alloy.

21. For Babu (2015: 83), this would perhaps be better phrased as a “world of texts” opposed to a “world of practice.”

Here, then, the purity 7 + 1/2 *māttu* and a measure of weight (4 *paṇaveṭai* of gold) are multiplied by one another. The product obtained is a number with no associated measuring unit, 4 x 15/2 = 30. The sum of measures of weight of the gold and silver (4 *paṇaveṭai* of gold + 1 *paṇaveṭai* of silver = 5 *paṇaveṭai*) is computed and then the division of the product by the sum (30/5) provides the "new *māttu*," 6. Here, then, one step of the process is not associated with a measuring unit, not unlike what is common in Sanskrit mathematical texts.

Further, among Tamil sources, the *Kaṇitanūḷ*, a text known in one manuscript at the Government Oriental Manuscript Library (in Chennai) seems to be closer to Sanskrit sources for its emphasis on computations with numbers than to the kind of practices found in the *Kaṇakkatikāram*. Here is a problem for the payment of wages:

> Finding the gold in 6 *māttu* for one who worked for five days, when one worked for fifteen days and gets 4 *paṇaveṭai* gold of 9 *māttu*. The last quantity of 9 *māttu* gold and its weight 4 are multiplied which is 36. This has to be multiplied by the number of days of the first, which is 5=180. Now divide this by the last given number of days which is 15, 180/15 = 12. Now divide this by the first *māttu* which is 2 *paṇaveṭai*.
>
> (Babu 2015: 52, quoting Subramaniam 1999: 226–29, verse 70–73)

We can recognize here a Rule of Five, a familiar tool of Sanskrit mathematical texts: To find the weight of gold w_n paid to somebody who worked for n days, for a payment in gold of refinement m_n, knowing that a work of x days, with a gold refinement of m_x is paid w_x, the procedure amounts to computing: $w_n = (nm_xw_x)/xm_n$. The computation carried out, first computing $nm_xw_x = 9 \text{ x } 4 \text{ x } 5=180$, and then dividing this sum first by x = 15 and then by $m_n = 4$, thus finding the result 2 *paṇaveṭai*, articulates well how the initial values each have a separate associated measuring unit, but the intermediate steps are numbers to which no distinct measure can be associated.

Here is an alloy problem, whose numerical values seem to echo one we have seen above:

> When a 8 *māttu* gold weighing 2 *paṇaveṭai* and another 7 *māttu* gold weighing 2 *paṇaveṭai* are melted, what would be the resulting *māttu*? The method is to multiply the *māttu* and *paṇaveṭai* of each gold and add them. That is (8 x 2) + (7 x 2) = 16 + 14 = 30. Add the weights, 2 *paṇaveṭai* + 2 *paṇaveṭai* = 4 *paṇaveṭai*. Now, divide the previous 30 by this 4 to get 7 1/2. Therefore, the resultant *māttu* gold will have a *māttu* of 7 1/2. (Babu 2015: 50–51, quoting Subramaniam 1999: 212, verse 58)

We recognize here the same process as the one found in the *Kaṇakkatikāram* and the *Līlāvatī*, in which to find the fineness of an alloy, the sum of the products of weights and purity is divided by the sum of weights ($V=\Sigma w_i v_i/\Sigma w_i$). The emphasis here is first on the application of a general abstract rule ("to multiply the *māttu* and *paṇaveṭai* of each gold and add them"), which is applied. It is striking that the elements of the process to which no measuring unit can be associated are not articulated, while those to which an interpretation in terms of measures can be made, are. The resulting value of *māttu* 7 1/2, is the same as in the *Kaṇakkatikāram*. Would these two texts be in dialogue with one another?

The recognition of this circulation of problems and practices, within the Tamil sources and with some of the Sanskrit canons, hints at the fact that probably these ways of computing and practicing mathematics were not separate geographically one from another. In his study of the local village schools in South-East Tamil Nadu, Babu notes that Brahmin children would go to funded Sanskrit schools, while other children would rather be schooled in the Tamil ones. Implicitly, there seems to have been a separate space for Sanskrit lore within Brahmin *maṭhas*. Babu (2015: 116) also shows how these worlds interacted: some teachers of the village schools knew Sanskrit mathematical texts. Translations are also known from the Sanskrit to the Tamil. There is a trope of exchange—when we come to the relations of Sanskrit with the vernaculars—in mathematics. Thus, in his pioneering essay on the topic, Sarma states:

> It is certain that these exchanges were never one-sided, i.e., from the “Great Tradition” of Sanskrit to the “Little Traditions” of regional languages. The two traditions were mutually complementary. While mathematical ideas and processes were systematized in Sanskrit manuals, the broader dissemination of these ideas took place in the regional languages. Conversely, Sanskrit has also absorbed much from the local traditions. (Sarma 2011: 221–22)

Implicitly, a social role is devoted to both tradition. Sanskrit is understood as the realm of the highbrow elite, while Tamil is used for the “dissemination” of the knowledge created in Sanskrit. In the cases we have found here, the Sanskrit texts seem to want to build a mathematical understanding and a more general mathematical context for the rules applied in the *Kaṇakkatikāram*.

Similarly, the fact that even in the seventeenth century computations could be carried out without using the decimal place value notation may explain why computations in Sanskrit texts also hint at methods not involving this notation.[22] Time and space prevent me from developing the point here, but it seems that a whole range of rules that were interpreted as expressing the commutativity, associativity, and distributivity of multiplication may, in fact, be read as providing general procedures when computing with numerals that do not use place-value notations.

A problem quoted by Sarma (which seems to have traveled widely from Europe to Japan) alludes both to how multiple exchanges could take place across language and social

22. Keller and Morice-Singh (forthcoming) treats of the use or not of decimal place value notations while observing techniques of multiplication in Sanskrit texts.

barriers and also to how the knowledge was kept mostly to the Brahmins, in an atmosphere not at all civil:[23]

> Fifteen Brahmins and fifteen thieves had to spend a dark night in an isolated temple of Durga. The goddess appeared in person at midnight and wanted to devour exactly fifteen persons, since she was hungry. The thieves naturally suggested that she should consume the fifteen plump Brahmins. But the clever Brahmins proposed that all the thirty would stand in a circle and Durga should eat each ninth person. The proposal was accepted by Durga and the thieves. So the Brahmins arranged themselves and the thieves in a circle, telling each one where to stand. Durga then counted out each ninth person and devoured him. When the fifteen were eaten, she was satiated and disappeared, and only Brahmins remained in the circle. The problem is, how did the Brahmins arrange themselves and the thieves in a circle? (Sarma 2011: 210, also quoted in Babu 2015: 1)

Conclusion: What speaks to ontological differences?

I borrow the concept of mathematical practices and culture from Karine Chemla, Renaud Chorlay, and David Rabouin (2016), and more largely to a group in Paris that reflects on these questions. Mathematical cultures in this understanding, which associates shared practices with shared values, are fundamentally plural. Of course language and local contexts come into play in ways that need to be described as well, but it is striking that sometimes across language and space people with the same occupations develop similar practices which can be very different from the practices and values of people sharing the same language but not the same occupation. From what we have seen in this essay, it is reasonable to believe that it was not two cultures of mathematics but many that have existed in the Indian subcontinent during its very rich, vast, and long history. If we do not know of many of these

23. For a more in-depth study of the problem and its circulations, see Sarma (1987).

cultures as of today, it is because they have been overlooked. Indeed, for a very long time, what counted as valuable was what was similar to present day mathematics on the one hand, and the practices of a cultivated elite on the other hand. If we come back to the situation of the Nagapattinam region of Tamil Nadu evoked in the beginning of this essay, the question will be first to understand the social contexts that these mathematics are an emanation of, the story that brought this situation into being. Second, if we consider this situation as testifying to contemporary and geographically close but differing mathematical cultures, we may want to raise questions that might help us think of how different these mathematical practices are: Does the adoption of one exclude the other? Can these mathematical practices cross-fertilize each other, can they produce a hybrid? Have they done so in the past? Could this serve as a criterion of ontological difference?

But in the context of South Asia today what would it mean to hold that different mathematical cultures have different mathematical ontologies? Those who brandish ontological differences today are those who want to see Hindu, Christian, and Muslim cultures as ontologically different. The current fundamentalist and nationalist government would like to promote the idea of Indian traditional sciences as alternatives to Western secular-Christian-Muslim science, very much insisting on the fundamental difference and power that come from exploring science with mystical and religious tools. In this very political atmosphere, let us leave the question of the existence of different mathematical ontologies open, but remember that there have been and still are different cultures of how we deal with values and numbers, cultures of measure, and dealings with shapes. Some are still alive today. They are studied by ethno-mathematics and should be thought of as a cultural heritage in danger that needs to be preserved.

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Appendix: Measuring Units

Here, I group and explain the different measuring units found in the extracts discussed in this essay.

A. Measuring Units in Sanskrit Sources

Measures of lengths and areas

lengths

24 *aṅgulas* (breadth of a finger) make a *hasta* (forearm, cubit).

areas

24 square *aṅgulas* make a square *hasta.*

The ratio is the same as in the linear case. Consequently, the square *aṅgula* cannot be thought of as being constructed from linear *aṅgulas*. Indeed, a square linear *hasta* should contain 24 x 24= 576 square linear *aṅgulas*. 24 is not a perfect square; consequently, this square *hasta* seems to be a theoretical unit.

Measures relating to gold

purity is measured not in *carats* but in *varṇa* (colors)

weight of gold involves small specialized measuring units such as the *gadyāṇaka*. The *māṣa* is a more standard weight. The text does not specify how they are related.

prices are given in *niṣkas*, a gold coin, in the *Līlāvatī.* The treatise provides a rule from which we know that there would have been 256 (16 x 16) *paṇas* in a gold *niṣka*.

B. Measuring Units in Tamil Sources

Measures of lengths and areas

lengths

kōl is the name of a measuring rod, and also of a unit of length.

areas

1 *kuḻi* is a square whose side is 1 *kōl*

Many relations of the *vēli* to the *kuḻi* are documented, probably the cholas imposed the following:

2000 *kuḻi* make a *vēli*

Measures relating to gold

fineness measured in *māttu*

weight measured in *kalañcu* and *paṇaveṭai*

coins are evoked through the *paṇam* here. Silver and coper punch-marked *paṇa*s are known to have been used as early as the Mauryan empire in South Asia (321 BCE–187 CE). In the *Līlāvatī*, a rule explains that there would have been 256 (16 x 16) *paṇas* in a gold *niṣka*.