Refined Neutrosophic Matrices Representations

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Abstract:

This paper studies the possibility of representing refined neutrosophic matrices by linear functions over a refined neutrosophic vector space.

Main Discussion

Definition 1:

Let V, W be two vector spaces over R, consider any three linear transformations $g, h, q: V \to W$. We define the corresponding full AH-refined linear transformation as follows:

$$f: V(I_1, I_2) \to w(I_1, I_2),$$

$f(x + yI_1 + zI_2) = g(x) + I_1[(g + h + q)(x + y + z) - (g + q)(x + z)] + I_2[(g + q)(x + z) - g(x)]$.

We denote it by $f = g + hI_1 + qI_2$

Theorem 2:

Let $f = g + hI_1 + qI_2: V(I_1, I_2) \to W(I_1, I_2)$ be any full AH-refined linear transformation, then f is linear by classical meaning.

Proof:

$\forall\, X = x + yI_1 + zI_2, Y = x_1 + y_1I_1 + z_1I_2 \in V(I_1, I_2)$ we have:

$$\begin{aligned} f(X + Y) &= f[(x + x_1) + I_1(y + y_1) + I_2(z + z_1)] \\ &= g(x + x_1) + I_1[(g + h + q)(x + x_1 + y + y_1 + z + z_1) - (g + q)(x + x_1 + z + z_1)] \\ &\quad + I_2[(g + q)(x + x_1 + z + z_1) - g(x + x_1)] \\ &= g(x) + g(x_1) \\ &\quad + I_1[(g + h + q)(x + y + z) + (g + h + q)(x_1 + y_1 + z_1) - (g + q)(x + z) \\ &\quad - (g + q)(x_1 + z_1)] + I_2[(g + q)(x + z) + (g + q)(x_1 + z_1) - g(x) - g(x_1)] \end{aligned}$$

$= \big[g(x) + I_1[(g + h + q)(x + y + z) - (g + q)(x + z)] + I_2[(g + q)(x + z) - g(x)]\big] + [g(x_1) + I_1[(g + h + q)(x_1 + y_1 + z_1) - (g + q)(x_1 + z_1)] + I_2[(g + h + q)(x_1 + z_1) - g(x_1)]$,

$$= f(X) + f(Y)$$

$\forall\, r = r_0 + r_1I_1 + r_2I_2 \in V(I_1, I_2)$, we have:

$$r \cdot X = r_0x + I_1[(r_0 + r_1 + r_2)(x + y + z) - (r_0 + r_2)(x + z)] + I_2[(r_0 + r_2)(x + z) - r_0x]$$

$f(r \cdot X) = g(r_0 x) + I_1[(g+h+q)[r_0 + r_1 + r_2)(x+y+z) - (g+q)[(r_0+r_2)(x+z)] + I_2[(g+q)[(r_0+r_2)(x+z)] - g(r_0 x)],$

Now we compute $r \cdot f(X)$:

$r \cdot f(X) = (r_0 + r_1 I_1 + r_2 I_2) \cdot [g(x) + I_1[(g+h+q)(x+y+z) - (g+q)(x+z)] + I_2[(g+q)(x+z) - g(x)],$
$= r_0\, g(x) + I_1[(r_0+r_1+r_2)(g+h+q)(x+y+z) - (r_0+r_2)(g+q)(x+z)] + I_2[(r_0+r_2) \cdot (g+q)(x+z) - r_0 g(x)] = f(rX).$

Definition 3:

Let $M = A + BI_1 + CI_2$ be a matrix over $R(I_1, I_2)$. We say that M is the matrix of $f = g + hI_1 + qI_2$iff $f(x + yI_1 + zI_2) = M \cdot (x + yI_1 + zI_2)$.

Theorem 4:

Let $f = g + hI_1 + qI_2 : V(I_1, I_2) \to W(I_1, I_2)$ be any full AH-refined linear transformation, then $M = A + BI_1 + CI_2$ is the matrix of f iff A is the matrix of g, B is the matrix of h, C is the matrix of q.

Proof:

Firstly, we suppose that A is the matrix of g, B is the matrix of h, C is the matrix of q, where $g, h, q: V \to W$ are classical linear transformations.

Consider the refined neutrosophic matrices $M = A + BI_1 + CI_2$, we have for each $x + yI_1 + zI_2 \in V(I_1, I_2)$:

$M(x + yI_1 + zI_2) = (A + BI_1 + CI_2)(x + yI_1 + zI_2) = A \cdot x + I_1[(A+B+C)(x+y+z) - (A+C)(x+z)] + I_2[(A+C)(x+z) - A \cdot x],$
$= g(x) + I_1[(g+h+q)(x+y+z) - (g+q)(x+z)] + I_2[(g+q)(x+z) - g(x)] = f(x + yI_1 + zI_2)$, hence M is the matrix of f.

Conversely, we assume that $M = A + BI_1 + CI_2$ is the matrix of f, we must prove that A is the matrix of g, B is the matrix of h, C is the matrix of q.

We have $f(x + yI_1 + zI_2) = M(x + yI_1 + zI_2)$, thus: $g(x) + I_1[(g+h+q)(x+y+z) - (g+q)(x+z)] + I_2[(g+q)(x+z) - g(x)] = Ax + I_1[(A+B+C)(x+y+Z) - (A+C)(x+z)] + I_2[(A+C)(x+z) - Ax]$

So that, $g(x) = Ax, (A+C)(x+z) = (g+q)(x+z), (A+B+C)(x+y+z) = (g+h+q)(x+y+z)$

This implies that A is the matrix of $gA + C$ us the matrix of $g + q$, thus C is the matrix of q and $A + B + C$ is the matrix of $g + h + q$, thus B is the matrix of h.

In order to prove that every refined neutrosophic matrix can be represented by a unique AH-refined linear transformation, we interduce the following algorithm to derive a basis of $V(I_1, I_2)$ from any classical basis of V.

Theorem 5: Let $S = \{V_1, V_2, \dots, V_n\}$ be a basis of V over R, then:

$$L_{i,j\,k} = \{V_i + (V_j - V_k)I_1 + (V_k - V_i)I_2\}$$

Is a basis of $V(I_1, I_2)$ over $R(I_1, I_2)$.

Theorem 6: Let $f: V(I_1, I_2) \to W(I_1, I_2)$ be a linear function, then f must be a full AH-refined linear transformation.

Example 7:

Now, we give an example to clarify the structure of the refined neutrosophic basis of a refined neutrosophic vector space basing on Theorem 5.

It is well known that the $R^2$is a vector space over the real field R. Consider the corresponding refined neutrosophic vector space
$R^2(I_1, I_2) = \{a + bI_1 + cI_2; a, b, c \in R^2\} = \{(x, y) + I_1(z, t) + I_2(m, n); m, n, z, t, x, y \in R\}$.

The basis of R^2 is the set $\{e_1 = (1,0), e_2 = (0,1)\}$. According to Theorem 5, the related refined neutrosophic basis of $R^2(I_1, I_2)$ is:

$v_1 = e_1 + I_1(e_1 - e_1) + I_2(e_1 - e_1), v_2 = e_1 + I_1(e_2 - e_1) + I_2(e_1 - e_1)$

$v_3 = e_1 + I_1(e_2 - e_2) + I_2(e_2 - e_1), v_4 = e_1 + I_1(e_1 - e_2) + I_2(e_2 - e_1),$

$v_5 = e_2 + I_1(e_1 - e_1) + I_2(e_1 - e_2), v_6 = e_2 + I_1(e_1 - e_2) + I_2(e_2 - e_2),$

$v_7 = e_2 + I_1(e_2 - e_1) + I_2(e_1 - e_2), v_8 = e_2 + I_1(e_2 - e_2) + I_2(e_2 - e_2).$

Example 8:

Now, we illustrate an example to clarify the representation of refined linear transformations by refined neutrosophic matrices.

Let $f: R^2(I_1, I_2) \to R(I_1, I_2); f[(x, y) + I_1(z, t) + I_2(m, n)] = f[(x + zI_1 + mI_2, y + tI_1 + nI_2)]$=

$2(x + zI_1 + mI_2) - (y + tI_1 + nI_2) = (2x - y) + (2z - t)I_1 + (2m - n)I_2.$

be a refined linear transformation, we have to write f as an AH-refined linear transformation.

By Theorem 6, we know that f must be equal to $g + hI_1 + qI_2$, where $g, h, q: R^2 \to R$.

Firstly, we have $g: R^2 \to R; g(x, y) = 2x - y$, and $(g + q)[(x, y) + (m, n)] - g(x, y) = 2m - n,$

This means that: $g(x + m, y + n) + q(x + m, y + n) - (2x - y) = 2m - n$, so that

$2(x + m) - (y + n) + q(x + m, y + n) - (2x - y) = 2m - n$, hence

$2m - n + q(x + m, y + n) = 2m - n, thus\ q(x + m, y + n) = 0\ for\ all\ x, y, m, n.$

This implies that $q(x, y) = 0$ is a zero map.

For the computing of h, we regard that: $(g + h + q)[(x, y) + (z, t) + (m, n)] - (g + q)[(x, y) + (m, n)] = 2z - t$.

So that: $(g + q + h)[(x + z + m, y + t + n)] - (g + q)[(x + m, y + n)] = 2z - t$. By remarking that

$g + q = g$, we get:

$(g + h)[(x + z + m, y + t + n)] - g[(x + m, y + n)] = 2z - t$, hence

$g(x + z + m, y + t + n) + h(x + z + m, y + t + n) - [2(x + m) - (y + n)] = 2z - t$,

This means that: $2(x + z + m) - (y + t + n) + h(x + z + m, y + t + n) - 2x - 2m + y + n = 2z - t$,

Thus $h(x + z + m, y + t + n) = 0 \; for \; all \; x, y, z, m, n, t$, i.e. h is a zero map.

By the previous discussion, we get that $f = g + 0.I_1 + 0.I_2$.

The matrix of $g \; is \; \begin{pmatrix} 2 \\ -1 \end{pmatrix}, the \; matrix \; of \; q \; and \; h \; is \; \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, thus the refined neutrosophic matrix of f is

$\begin{pmatrix} 2 \\ -1 \end{pmatrix} + I_1 \begin{pmatrix} 0 \\ 0 \end{pmatrix} + I_2 \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} = M.$

Now, we check that M is refined neutrosophic matrix of f by the following computing:

$M.[(x, y) + I_1(z, t) + I_2(m, n)] = M.[(x + zI_1 + mI_2, y + tI_1 + nI_2)] = 2(x + zI_1 + mI_2) - (y + tI_1 + nI_2) = f(x + zI_1 + mI_2, y + tI_1 + nI_2)$.

Example 9:

In example 8, we showed how a refined neutrosophic linear transformation can be turned into a refined neutrosophic matrix, now we do the converse. We illustrate an example to show how a refined neutrosophic matrix can be turned into a refined neutrosophic linear transformation.

Let $M = \begin{pmatrix} 1 & I_2 \\ 3 - I_1 + 2I_2 & 2I_1 \end{pmatrix}$ be a refined neutrosophic matrix. M can be written in the following form:

$M = A + BI_1 + CI_2 = \begin{pmatrix} 1 & 0 \\ 3 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ -1 & 2 \end{pmatrix} I_1 + \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix} I_2.$

M can be represented by a unique refined neutrosophic linear transformation

$f: R^2(I_1, I_2) \to R^2(I_1, I_2); f[(x + zI_1 + mI_2, y + tI_1 + nI_2)] = M.[(x + zI_1 + mI_2, y + tI_1 + nI_2)] =$

$(x + I_1(z + t) + I_2(m + y + n), 3x \; + (4z - x + 2y + 2t + 2n - m)I_1 + (2x + 5m)I_2)$.

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