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► To cite this version:

Shuai Ma, Ruixin Yang, Xiong Deng, Xintong Ling, Xun Zhang, et al.. Spectral and Energy Efficiency of ACO-OFDM in Visible Light Communication Systems. IEEE Transactions on Wireless Communications, inPress. hal-03244917

HAL Id: hal-03244917

<https://hal.science/hal-03244917>

Submitted on 17 Jul 2021

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Journal:	<i>IEEE Transactions on Wireless Communications</i>
Manuscript ID	Draft
Manuscript Type:	Original Transactions Paper
Date Submitted by the Author:	n/a
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Keyword:	

Spectral and Energy Efficiency of ACO-OFDM in Visible Light Communication Systems

Shuai Ma, Ruixin Yang, Xiong Deng, Xintong Ling, Xun Zhang, Fuhui Zhou,
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Abstract

In this paper, we study the spectral efficiency (SE) and energy efficiency (EE) of asymmetrically clipped optical orthogonal frequency division multiplexing (ACO-OFDM) for visible light communication (VLC). Firstly, we derive the achievable rates for Gaussian distributions inputs and practical finite-alphabet inputs. Then, we investigate the SE maximization problems subject to both the total transmit power constraint and the average optical power constraint with the above two inputs, respectively. By exploiting the relationship between the mutual information and the minimum mean-squared error, an optimal power allocation scheme is proposed to maximize the SE with finite-alphabet inputs. To reduce the computational complexity of the power allocation scheme, we derive a closed-form lower bound of the SE. Also, considering the quality of service, we further tackle the non-convex EE maximization problems of ACO-OFDM with the two inputs, respectively. The problems are solved by the proposed Dinkelbach-type iterative algorithm. In each iteration, the interior point algorithm is applied to obtain the optimal power allocation. The performance of the proposed power allocation schemes for the SE and EE maximization are validated through numerical analysis.

Index Terms

ACO-OFDM, energy efficiency, spectral efficiency, visible light communications.

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I. INTRODUCTION

Traditional radio frequency (RF) communications are facing the problem of spectrum crunch because of the exponential increase in the demand for wireless data traffic [1], [2]. Besides, the tremendous wireless devices consume more than 3% of the global energy [3], [4], and lead to about 5% of the total CO₂ emissions worldwide by 2020 [5]–[7]. Therefore, both spectral and energy resources are severely limited for next generation wireless communications. Facilitated by the low-cost and widely installed lighting infrastructure with light emitting diodes (LEDs), visible light communication (VLC) has emerged as a promising green indoor communication solution enabling simultaneous illumination and wireless data transmission. Owing to its inherent advantages, such as abundant license-free spectrum, high security, and no interference to existing RF-based systems, VLC systems are a compelling supplementary to RF systems for realizing high-speed wireless data transmissions.

Despite the promising gains brought VLC technologies, serious inter-symbol interference (ISI) creates a system performance bottleneck due to the existence of multipath in high-data rate VLC systems. In practice, orthogonal frequency-division multiplexing (OFDM) [8] is an effective solution to trackle ISI in RF-based systems. However, as VLC systems exploit intensity modulation and direct detection (IM/DD) schemes for communication, information of VLC is represented by light intensity and thus transmitted signals should be real-valued and nonnegative. Thus, conventional RF-based OFDM techniques cannot directly apply to VLC systems. To mitigate the ISI issue, asymmetrically clipped optical OFDM (ACO-OFDM) [9], [10] and direct current biased optical OFDM (DCO-OFDM) [11], [12] have been proposed for VLC systems. To generate nonnegative transmitted signals, ACO-OFDM eliminates the negative component of signals, while DCO-OFDM adds a direct current (DC) bias and then clips the negative parts of signals by setting them to zero [13]. Moreover, ACO-OFDM transmits data symbols only via odd indexed subcarriers, whereas DCO-OFDM transmits data symbols exploiting all the subcarriers. Compared with DCO-OFDM, ACO-OFDM can generally achieve a lower bit-error-rate (BER) for identical QAM modulation orders such as in [14], [15]. However, due to only half of the subcarriers to carry information, the spectral efficiency (SE) of ACO-OFDM is generally lower than that of DCO-OFDM from moderate to high signal-to-noise ratio (SNR) [16].

Recently, various power allocation schemes have been proposed to improve the SE of ACO-OFDM VLC systems. For example, under average optical power constraint, the conventional

water-filling power allocation scheme can improve the information rate of ACO-OFDM substantially [10]. In [17], the achievable rates of ACO-OFDM and filtered ACO-OFDM (FACO-OFDM) were analyzed with both optical power and bandwidth constraints. Besides, by taking into account both average optical power and dynamic optical power constraints, both the error vector magnitude (EVM) and achievable data rates of the DCO-OFDM and ACO-OFDM systems were analyzed in [18] showing that ACO-OFDM can achieve the lower bound of the EVM. In [19], two upper bounds of channel capacity for the intensity modulated direct detection (IM/DD) optical communication systems were derived based on an exponential input distribution and clipped Gaussian input distribution respectively, and a closed-formed channel capacity of ACO-OFDM. However, all of them only considered the average optical power constraint. Then, a more detailed description of the problem was given in [10] with an electrical power limit or both optical power limit and input power constraint. For the latter scenario, [10] proved that if the real and imaginary components of each odd frequency IFFT input are independent random variables with circular symmetry, they must follow a zero mean Gaussian distribution and the outputs of the IFFT are strict sense stationary. Then, a closed-form information rate was derived to satisfy above conditions. However, the more general question of the information rate of an ACO-OFDM system only with limited average optical power remains an intractable problem. Also, subject to a given a target BER requirement, adaptive modulation schemes were investigated in [20] to maximize the SE of DCO-OFDM, ACO-OFDM, and single carrier frequency-domain equalization (SC-FDE) systems, respectively. In spite of the fruitful research in the literature, the aforementioned studies were based on the assumption that the input signal follows Gaussian distribution. Although Gaussian distribution inputs can achieve the channel capacity under average electrical power constraints, the optimal distribution with optical power constraints is still unknown. In fact, practical input signals are often based on discrete constellation schemes, such as pulse amplitude modulation (PAM), quadrature amplitude modulation (QAM), and phase shift keying (PSK). Applying power allocation schemes based on Gaussian distribution inputs to signals with practical finite-alphabet inputs may cause serious performance loss [21]. So far, the SE of ACO-OFDM with finite-alphabet inputs has been rarely considered in the literature. Therefore, it is necessary to design an optimal power allocation scheme to unlock the potential of ACO-OFDM systems.

In addition to improving the SE, achieving high energy efficiency (EE) is also critical for ACO-OFDM VLC systems, which is usually defined as a ratio of the achievable rate to the total power consumption [22]. In fact, the improved SE does not come for free. In particular, the

improvement is always achieved at the expense of increased energy cost. Unfortunately, most of the aforementioned VLC research [23] aimed at improving SE, but omitted the EE of ACO-OFDM systems. Recently, there are some works started focusing on the EE issue in VLC systems. By the joint design of the cell structure and the system level power allocation, an amorphous structure of ACO-OFDM VLC systems can achieve a higher EE than that of the conventional cell structures [24]. To ensure the quality of service (QoS) with affordable energy, the EE of the conventional and hybrid OFDM-based VLC modulation schemes was investigated in [25]. However, existing studies of ACO-OFDM's EE [24], [25] are based on Gaussian distribution inputs. As previously mentioned, Gaussian distribution inputs are difficult to generate in practice. Indeed, practical inputs are always finite-alphabet inputs, which has been less commonly studied in literature. Thus, there is an emerge need for the study of EE of ACO-OFDM VLC systems with finite-alphabet inputs.

In this study, we propose the optimal power allocation scheme to maximize the SE and the EE of ACO-OFDM VLC systems with Gaussian distribution inputs and finite-alphabet inputs, respectively. The main contributions of this paper are summarized as follows:

- We systematically analyze the signal processing module of a typical ACO-OFDM VLC system. Based on the frequency domain analysis, we first derive achievable rates of the considered system admitting finite-alphabet inputs from the perspective of practical modulation. Additionally, for both cases of the Gaussian distribution inputs and finite-alphabet inputs, we develop the corresponding optical power constraints for ACO-OFDM VLC systems.
- Under both the total transmit power constraint and the optical power constraint, two optimal power allocation schemes are proposed to maximize the SE of the ACO-OFDM system with Gaussian distribution inputs and finite-alphabet inputs, respectively. Specifically, for Gaussian distribution inputs, we show that the water-filling-based power allocation scheme can maximize the SE. Similarly, for finite-alphabet inputs, we derive an optimal power allocation scheme to achieve the maximum SE by exploiting the Lagrangian method, Karush-Kuhn-Tucker (KKT) conditions, and the relationship between the mutual information and the minimum mean-squared error (MMSE) [26].
- The optimal power allocation scheme for finite-alphabet inputs lacks closed-form expressions and involves complicated computations of MMSE. To reduce the computational complexity, we first derive a closed-form lower bound for the achievable rate. Then, based on the proposed lower bound, we develop a suboptimal power allocation scheme to maximize the

SE under both the total transmit power constraint and the average optical power constraint.

- We propose an explicit EE expression with Gaussian distribution inputs and finite-alphabet inputs, respectively. Moreover, under the constraint of maximum transmission power and the minimum data rate requirement, the non-convex problem of maximizing EE is investigated. This problem is solved by applying the Dinkelbach-type algorithm and the interior point algorithm. Finally, the relationship between the SE and the EE of the ACO-OFDM system is unveiled.

The rest of this paper is organized as follows. The system model of ACO-OFDM is presented in Section II. The SE of ACO-OFDM system is shown in Section III. The EE of ACO-OFDM system is studied in Section IV and the simulation results are presented in Section V. Finally, the conclusions are drawn in Section VI.

Notations: Boldfaced lowercase and uppercase letters represent vectors and matrices, respectively. Expected value of a random variable z is denoted by $\mathbb{E}\{z\}$. $(\cdot)^*$ represents conjugate transformation. $[x]^+$ denotes $\max\{x, 0\}$. $\text{Re}(\cdot)$ denotes the real part of its argument. $\frac{\partial f(\cdot)}{\partial x}$ represents the partial derivative operation on x of function $f(\cdot)$. Given a variable y , $\mathbb{E}\{z|y\}$ represents the conditional mean of z for given y . $\min\{x, y\}$ represents the minimum value between x and y . $I(X; Y)$ represents the mutual information of X and Y . A complex-valued circularly symmetric Gaussian distribution with mean μ and variance σ^2 is denoted by $\mathcal{CN}(\mu, \sigma^2)$. A real-valued Gaussian distribution with mean μ and variance σ^2 is denoted by $\mathcal{N}(\mu, \sigma^2)$.

II. SYSTEM MODEL

Consider an ACO-OFDM VLC system with total $2N$ subcarriers, as shown in Fig. 1 [15], where the signals are only transmitted via the odd indexed subcarriers. The information bit stream is first converted to parallel sub-streams by a serial-to-parallel (S/P) converter. Then, they are modulated by an M -QAM scheme. After applying the inverse fast Fourier transform (IFFT) and zero clipping on the modulated symbols, signal \hat{x}_k is non-negative. Then, the signal passes through an analog to digital converter (D/A), where the digital signal \hat{x}_k is converted to an analog signal. After that, the analog signal is emitted through visible light by an LED. In particular, by exploiting the IM/DD scheme, the transmitted information of the VLC system is represented by the signal intensity, which is real and non-negative. At the receiver, the received visible light is transformed into an analog electrical signal by a photo detector (PD) and then converted to a digital signal by an analog to digital converter (A/D). After applying the fast

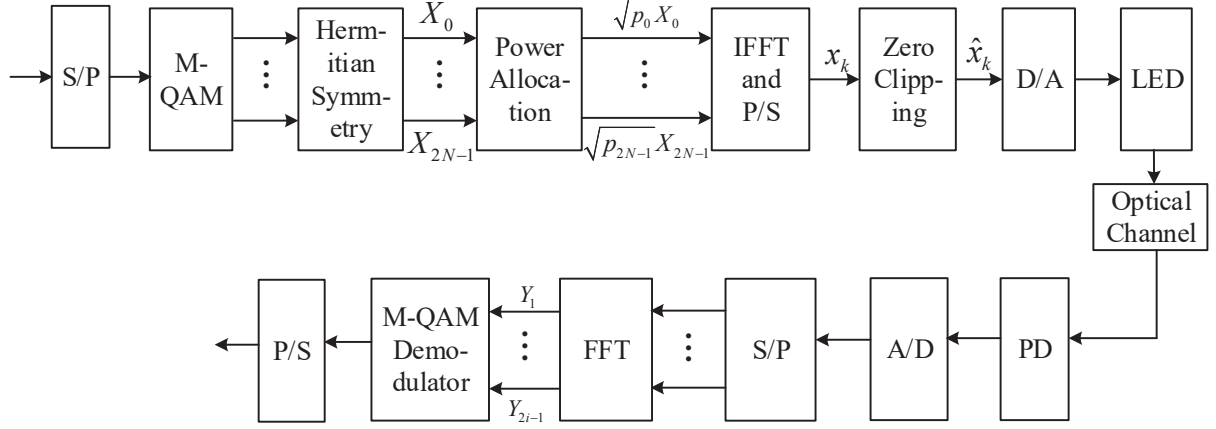


Fig. 1: A block diagram of an ACO-OFDM VLC system.

Fourier transform (FFT) on the digitalized signal, demodulation is performed at a demodulator to convert the received M -QAM symbols to bit streams.

A. Signal Model

In this section, we discuss the mathematical details of the considered system. At the transmitter, the raw data bit stream is going through the modulation. Let X_k denotes the modulated signal on the k th subcarrier, and p_k denotes the allocated power on the k th subcarrier, $k = 0, \dots, 2N - 1$. Note that to ensure real output values at the IFFT, the input of the IFFT module should satisfy Hermitian symmetry, i.e.,

$$\begin{cases} X_{2i} = X_{2(N-i)-2} = 0, \\ X_{2i+1} = X_{2(N-i)-1}^*, \end{cases} \quad (1)$$

where $i = 0, \dots, N/2 - 1$, and X_{2i-1} is the normalized unit-power input, i.e., $\mathbb{E} \{|X_{2i-1}|^2\} = 1$. According to (1), the power allocation of subcarriers should satisfy

$$\begin{cases} p_{2i} = p_{2(N-i)-2} = 0, \\ p_{2i+1} = p_{2(N-i)-1} \geq 0, \end{cases} \quad (2)$$

where $i = 0, \dots, N/2 - 1$.

After the IFFT operation, the time domain signal x_k is given as

$$x_k = \text{IFFT} \{ \sqrt{p_i} X_i \} \quad (3a)$$

$$= \frac{1}{\sqrt{2N}} \sum_{i=0}^{2N-1} \sqrt{p_i} X_i \exp \left(j \frac{\pi k i}{N} \right) \quad (3b)$$

$$= \sqrt{\frac{2}{N}} \sum_{i=1}^{N/2} \sqrt{p_{2i-1}} \text{Re} \left(X_{2i-1} \exp \left(j \frac{\pi k (2i-1)}{N} \right) \right), \quad k = 0, \dots, 2N-1. \quad (3c)$$

According to (3c), the obtained time domain signal satisfies antisymmetry as follows

$$x_k = -x_{k+N}, \quad (4)$$

where $k = 1, \dots, N$. Since the transmitted signal should be nonnegative, the negative signals are removed by the clipping process such that

$$\hat{x}_k = \begin{cases} x_k & x_k \geq 0; \\ 0 & \text{otherwise.} \end{cases} \quad (5)$$

Let P denote the total electrical transmit power, i.e., $\sum_{k=0}^{2N-1} \mathbb{E} \{ \hat{x}_k^2 \} \leq P$. Combining the clipping process and Parseval's theorem [11], [27], we have $\sum_{i=0}^{2N-1} p_i = \sum_{k=0}^{2N-1} \mathbb{E} \{ x_k^2 \} = 2 \sum_{k=0}^{2N-1} \mathbb{E} \{ \hat{x}_k^2 \}$.

Based on (2), the electrical transmit power constraint can be rewritten as $\sum_{i=1}^{N/2} p_{2i-1} \leq P$. For the consideration of human eye safety, the optical power of VLC signals is generally restricted [28]–[32]. Let P_o represent the maximum optical power threshold. The average optical power should satisfy¹

$$\mathbb{E} \{ \hat{x}_k \} \leq P_o. \quad (6)$$

B. Channel Model

Generally, the VLC channel is characterized by a line-of-sight (LOS) link along with multiple reflections of the light from surrounding objects, such as walls, floor, and windows. In this study, we adopt the commonly used frequency-domain VLC channel model [33], which is not restricted to a finite order of reflections.

¹Due to different distributions, the signals with the same optical power may have different electrical powers.

Let H_i denotes the channel gain of the i th subcarrier, which includes both the LOS link and the diffuse links as follows

$$H_i = H_{L,i} + H_{D,i}, \quad (7)$$

where $H_{L,i}$ is the gain of the LOS link and $H_{D,i}$ is the gain of the diffuse links, $i = 0, \dots, 2N - 1$.

The LOS link $H_{L,i}$ is expressed as

$$H_{L,i} = g_L e^{-j2\pi f_i \tau}, \quad (8)$$

where g_L is the generalized Lambertian radiator [34], f_i denotes the frequency of the i th subcarrier, τ is the signal propagation delay between the transmitter and receiver with $\tau = d/c$, d is the distance between the transmitter and receiver, and c is the speed of light, $i = 0, \dots, 2N - 1$.

The generalized Lambertian radiator g_L can be expressed as

$$g_L = \begin{cases} \frac{(m+1)A_r \cos(\varphi)}{2\pi d^2} \cos^m(\theta) T(\varphi) G(\varphi) & 0 \leq \varphi \leq \Psi, \\ 0 & \text{otherwise,} \end{cases} \quad (9)$$

where m is the order of Lambertian emission, i.e., $m = -\ln 2 / \ln(\cos \Phi_{1/2})$, $\Phi_{1/2}$ is the half power angle; A_r is the effective detector area of the PD receiver; φ and θ are, respectively, the incidence and irradiance angle from the LED to the PD; $T(\varphi)$ and $G(\varphi)$ are the optical filter gain and the concentrator gain of the receiver, respectively; Ψ represents the field-of-view (FOV) of the receiver.

On the other hand, the gain of the diffuse links $H_{D,i}$ is given by [35]

$$H_{D,i} = \frac{\eta_D}{1 + j2\pi\tau f_i}, \quad (10)$$

where η_D is the power efficiency of the diffuse signal. Thus, the relationship between the time domain channel response and the channel gain in subcarriers can be described as $H(f_i) = \int_{-\infty}^{\infty} h(t) e^{-j2\pi f_i t} dt$.

C. Performance Metrics

In practice, the signals are transmitted from an LED through an optical channel. At the receiver, it performs FFT to obtain the frequency-domain modulated information. However, due to the zero clipping, the amplitude of the frequency domain signal at the receiver is half of that at the

transmitter [36]. Let Y_{2i-1} denote the signals received in the frequency-domain at the $(2i-1)$ th subcarrier, which is given by

$$Y_{2i-1} = \frac{1}{2} H_{2i-1} \sqrt{p_{2i-1}} X_{2i-1} + Z_{2i-1}, \quad (11)$$

where the coefficient $\frac{1}{2}$ exists since only half subcarriers are adopted to transmit information, Z_{2i-1} is the additive white Gaussian noise (AWGN) with zero-mean, i.e., $Z_{2i-1} \sim \mathcal{CN}(0, W\sigma^2)$, $i = 1, \dots, N$, and σ^2 represents the noise power spectral density, W represents the bandwidth of each subcarrier.

Let $R_{2i-1}(\{p_{2i-1}\})$ and R_{ACO} denote the rate of the $(2i-1)$ th subcarrier and the total rate of the ACO-OFDM system, respectively, which are given by

$$R_{2i-1}(\{p_{2i-1}\}) = I(X_{2i-1}; Y_{2i-1}), \quad (12a)$$

$$R_{\text{ACO}} = \sum_{i=1}^{N/2} R_{2i-1}(\{p_{2i-1}\}), \quad (12b)$$

respectively. Then, the SE of the ACO-OFDM VLC system is defined as the ratio of achievable data rate to the total bandwidth, which can be expressed as

$$\text{SE}(\{p_{2i-1}\}) = \frac{\sum_{i=1}^{N/2} R_{2i-1}(\{p_{2i-1}\})}{2NW}, \quad (13)$$

where W denotes the bandwidth of each subcarrier. At the same time, the EE of the ACO-OFDM VLC system is defined as the ratio of the capacity to the total power consumption, which can be expressed as

$$\text{EE}(\{p_{2i-1}\}) = \frac{\sum_{i=1}^{N/2} R_{2i-1}(\{p_{2i-1}\})}{\sum_{i=1}^{N/2} p_{2i-1} + P_c}, \quad (14)$$

where $\sum_{i=1}^{N/2} p_{2i-1}$ represents the total electrical power consumption of all the subcarriers and P_c denotes the total circuit power consumption of the whole system.

III. SPECTRAL EFFICIENCY OF ACO-OFDM

In this section, we aim to maximize the SE of the ACO-OFDM system under the electrical transmit power constraint and taking into account a practical average optical power constraint.

The considered problem can be mathematically formulated as follows:

$$\underset{\{p_{2i-1}\}}{\text{maximize}} \quad \frac{\sum_{i=1}^{N/2} R_{2i-1}(\{p_{2i-1}\})}{2NW} \quad (15a)$$

$$\text{s.t.} \quad \mathbb{E}\{\hat{x}_k\} \leq P_o, \quad (15b)$$

$$\sum_{i=1}^{N/2} p_{2i-1} \leq P, \quad (15c)$$

$$p_{2i-1} \geq 0, \quad i = 1, \dots, N/2. \quad (15d)$$

In the following, we will investigate the SE maximization problem (15) for the considered ACO-OFDM system with Gaussian distribution inputs and finite-alphabet inputs, respectively.

A. Gaussian Distribution Inputs

Assume that the input X_{2i-1} follows independent complex Gaussian distribution, i.e., $X_{2i-1} \sim \mathcal{CN}(0, 1)$. According to the IFFT operation (3c), the time domain signal x_k also follows Gaussian distribution, i.e., $x_k \sim \mathcal{N}\left(0, \frac{1}{N} \sum_{i=1}^{N/2} p_{2i-1}\right)$. Furthermore, based on the relationship in (5), the average optical power is given by

$$\mathbb{E}\{\hat{x}_k\} = \frac{1}{2} \mathbb{E}\{|x_k|\} = \frac{1}{2} \int_0^\infty x_k \frac{\sqrt{2}}{\sqrt{\pi}\sigma_s} e^{-\frac{x_k^2}{2\sigma_s^2}} dx_k = \sqrt{\frac{1}{2\pi N} \sum_{i=1}^{N/2} p_{2i-1}}, \quad (16)$$

where $\sigma_s^2 = \frac{1}{N} \sum_{i=1}^{N/2} p_{2i-1}$. Additionally, by substituting (16) into (15b), the average optical power constraint can be reformulated as

$$\sum_{i=1}^{N/2} p_{2i-1} \leq 2N\pi P_o^2. \quad (17)$$

According to the Shannon theorem [37], the achievable rate of Gaussian distribution inputs $R_G(\{p_{2i-1}\})$ is given by

$$R_G(\{p_{2i-1}\}) = W \log_2 \left(1 + \frac{p_{2i-1} |H_{2i-1}|^2}{4\sigma^2 W} \right). \quad (18)$$

Then, the SE of the Gaussian distribution inputs $\text{SE}_G(\{p_{2i-1}\})$ can be expressed as

$$\text{SE}_G(\{p_{2i-1}\}) = \frac{\sum_{i=1}^{N/2} \log_2 \left(1 + \frac{p_{2i-1}|H_{2i-1}|^2}{4\sigma^2 W} \right)}{2N}. \quad (19)$$

Thus, the SE maximization problem with the Gaussian distribution inputs can be rewritten as

$$\underset{\{p_{2i-1}\}}{\text{maximize}} \quad \text{SE}_G(\{p_{2i-1}\}) \quad (20a)$$

$$\text{s.t.} \quad \sum_{i=1}^{N/2} p_{2i-1} \leq \min \{P, 2N\pi P_o^2\}, \quad (20b)$$

$$p_{2i-1} \geq 0, \quad i = 1, \dots, N/2. \quad (20c)$$

Problem (20) is a convex optimization problem and satisfies the Slater's constraint qualification [38]. The problem in (20) can be solved by applying classical convex optimization approaches. To this end, we first need the Lagrangian function of (20), which is given as

$$\mathcal{L}_G = \frac{\sum_{i=1}^{N/2} \log_2 \left(1 + \frac{p_{2i-1}|H_{2i-1}|^2}{4\sigma^2 W} \right)}{2N} - \mu \left(\sum_{i=1}^{N/2} p_{2i-1} - \min \{P, 2N\pi P_o^2\} \right), \quad (21)$$

where $\mu \geq 0$ is the Lagrange multiplier associated with constraint (20b). By setting the differential function to 0, i.e., $\frac{\partial \mathcal{L}_G}{\partial p_{2i-1}} = 0$, the optimal p_{2i-1} is given by

$$p_{2i-1} = \left[\frac{1}{2N\mu \ln 2} - \frac{4\sigma^2 W}{|H_{2i-1}|^2} \right]^+. \quad (22)$$

In fact, (22) is known as the classical water-filling solution and the optimal μ can be found by the conventional gradient method or the epsilon method [38], [39].

B. Finite-alphabet Inputs

In practice, typical inputs are always based on discrete signaling constellations, such as M -PSK or M -QAM, rather than the ideal Gaussian signals. In this section, we assume that the inputs are drawn from discrete constellations set $\{X_{2i-1,k}\}_{k=1}^M$ with cardinality M , where $X_{2i-1,k}$ is a

constellation point of the $(2i - 1)$ th subcarrier. The achievable rate $R_F(\{p_{2i-1}\})$ is given by [21]

$$R_F(\{p_{2i-1}\}) = I_{2i-1}(X_{2i-1}; Y_{2i-1}) \quad (23)$$

$$= W \left(\log_2 M - \frac{1}{\ln 2} \right) - \sum_{n=1}^M \frac{W}{M} \mathbb{E}_Z \left\{ \log_2 \sum_{k=1}^M \exp(-d_{n,k}) \right\}, \quad (24)$$

where $I_{2i-1}(X_{2i-1}; Y_{2i-1})$ is the achievable mutual information over the $(2i - 1)$ th channel, $d_{n,k} = \frac{(\frac{1}{2} H_{2i-1} \sqrt{p_{2i-1}} (X_{2i-1,n} - X_{2i-1,k}) + Z_{2i-1})^2}{\sigma^2 W}$ is a measure of the difference between input constellation points $X_{2i-1,n}$ and $X_{2i-1,k}$, $\mathbb{E}_Z\{\cdot\}$ is the expectation of the noise Z_{2i-1} . Note that $R_F(\{p_{2i-1}\})$ is a concave function with respect to the power allocation p_{2i-1} [21], [40].

According to (5), the average optical power of transmitted signals is given as

$$\mathbb{E}\{\hat{x}_k\} = \frac{1}{2} \mathbb{E}\{|x_k|\} \leq \frac{1}{2\sqrt{2N}} \sum_{i=1}^{2N-1} \mathbb{E}\left\{\left|\sqrt{p_i} X_i \exp\left(j \frac{\pi k i}{N}\right)\right|\right\} = \frac{1}{\sqrt{2N}} \sum_{i=1}^N \sqrt{p_i} \mathbb{E}\{|X_i|\}, \quad (25)$$

where the inequality holds due to $\left|\sum_i a_i\right| \leq \sum_i |a_i|$, the value of $\mathbb{E}\{|X_i|\}$ depends on the specific modulation schemes. Furthermore, substituting (25) into (6), the average optical power constraint is given as

$$\frac{1}{\sqrt{2N}} \sum_{i=1}^{N/2} \sqrt{p_{2i-1}} \mathbb{E}\{|X_{2i-1}|\} \leq P_o. \quad (26)$$

Based on the inequality $\left(\sum_{i=1}^n a_i\right)^2 \leq n \left(\sum_{i=1}^n a_i^2\right)$ [41], where $a_i \geq 0$, the average optical power constraint (26) can be restricted as

$$\sum_{i=1}^{N/2} p_{2i-1} \leq \frac{4P_o^2}{\mathbb{E}^2\{|X_{2i-1}|\}}. \quad (27)$$

In other words, (27) is known as a safe approximation of (26) because the left side of (26) is replaced by its upper bound. After adopting the optimization algorithm, the optical power consumption would not exceed P_o . Then, the SE of finite-alphabet inputs $\text{SE}_F(\{p_{2i-1}\})$ can be expressed as

$$\text{SE}_F(\{p_{2i-1}\}) = \frac{\sum_{i=1}^{N/2} R_F(\{p_{2i-1}\})}{2NW}. \quad (28)$$

Thus, the optimal power allocation problem (15) can be reformulated as the constellation-constrained mutual information maximization problem which can be expressed as follows

$$\underset{\{p_{2i-1}\}}{\text{maximize}} \quad \text{SE}_F(\{p_{2i-1}\}) \quad (29a)$$

$$\text{s.t.} \quad \sum_{i=1}^{N/2} p_{2i-1} \leq \varepsilon, \quad (29b)$$

$$p_{2i-1} \geq 0, \quad i = 1, \dots, N/2, \quad (29c)$$

where $\varepsilon \triangleq \min \left\{ P, \frac{4P_o^2}{\mathbb{E}^2\{|X_{2i-1}|\}} \right\}$.

The lack of a closed-form expression for the objective function (29a) complicates its solution development. To address this difficulty, we aim to derive the optimal power allocation scheme for problem (15) by exploiting the relationship between the mutual information and MMSE [42].

To this end, we first derive the Lagrangian function of problem (29) which is given by

$$\mathcal{L}_F = - \sum_{i=1}^{N/2} R_F(\{p_{2i-1}\}) + \lambda \left(\sum_{i=1}^{N/2} p_{2i-1} - \varepsilon \right), \quad (30)$$

where $\lambda \geq 0$ is the Lagrange multiplier corresponding to constraint (29b).

Furthermore, the KKT conditions of problem (29) can be expressed as

$$- \frac{\partial R_F(\{p_{2i-1}\})}{\partial p_{2i-1}} + \lambda = 0, \quad (31a)$$

$$\lambda \left(\sum_{i=1}^{N/2} p_{2i-1} - \varepsilon \right) = 0, \quad (31b)$$

$$\sum_{i=1}^{N/2} p_{2i-1} - \varepsilon \leq 0, \quad (31c)$$

$$\lambda \geq 0, \quad p_{2i-1} \geq 0, \quad i = 1, \dots, N/2. \quad (31d)$$

According to [42], the relationship between the mutual information and the MMSE of the $(2i-1)$ th subcarrier is given by

$$\frac{\partial}{\partial \text{SNR}} I_{2i-1}(X_{2i-1}; Y_{2i-1}) = \text{MMSE}_{2i-1}(\text{SNR}), \quad (32)$$

where $\text{MMSE}_{2i-1}(\text{SNR}) = \mathbb{E} \left\{ \left| X_{2i-1} - \hat{X}_{2i-1} \right|^2 \right\}$ is the MMSE of X_{2i-1} , and \hat{X}_{2i-1} is condi-

ditional expectation of X_{2i-1} , i.e.,

$$\hat{X}_{2i-1} = \mathbb{E} \left\{ X_{2i-1} \middle| Y_{2i-1} = \frac{1}{2} H_{2i-1} \sqrt{p_{2i-1}} X_{2i-1} + Z_{2i-1} \right\}. \quad (33)$$

Combining (23) and (32), the differential function of $R_F(\{p_{2i-1}\})$ can be written as

$$\frac{\partial R_F(\{p_{2i-1}\})}{\partial p_{2i-1}} = \frac{|H_{2i-1}|^2}{4\sigma^2 W} \text{MMSE}_{2i-1} \left(\frac{|H_{2i-1}|^2}{4\sigma^2 W} p_{2i-1} \right). \quad (34)$$

By substituting (34) into (31a), we have

$$\frac{|H_{2i-1}|^2}{4\sigma^2 W} \text{MMSE}_{2i-1} \left(\frac{|H_{2i-1}|^2}{4\sigma^2 W} p_{2i-1} \right) = \lambda. \quad (35)$$

Then, solving (35) for the power allocation p_{2i-1} yields

$$p_{2i-1} = \frac{4\sigma^2 W}{|H_{2i-1}|^2} \text{MMSE}_{2i-1}^{-1} \left(\frac{4\sigma^2 W}{|H_{2i-1}|^2} \lambda \right), \quad (36)$$

where $\text{MMSE}_{2i-1}^{-1}(\cdot)$ is the inverse function of $\text{MMSE}_{2i-1}(\cdot)$ with the domain $[0, 1]$ and $\text{MMSE}_{2i-1}^{-1}(1) = 0$ [26].

Therefore, for the considered ACO-OFDM system, the optimal power allocation scheme of (29) is given by

$$p_{2i-1}^* = \begin{cases} \frac{4\sigma^2 W}{|H_{2i-1}|^2} \text{MMSE}_{2i-1}^{-1} \left(\frac{4\sigma^2 W}{|H_{2i-1}|^2} \lambda \right), & 0 < \lambda \leq \frac{|H_{2i-1}|^2}{4\sigma^2 W}; \\ 0, & \text{otherwise.} \end{cases} \quad (37)$$

The dual variable λ in (37) is the solution of the following equation

$$\sum_{i=1}^{N/2} \frac{4\sigma^2 W}{|H_{2i-1}|^2} \text{MMSE}_{2i-1}^{-1} \left(\frac{4\sigma^2 W}{|H_{2i-1}|^2} \lambda \right) = \varepsilon, \quad (38)$$

which can be obtained by a simple bisection method as listed in Algorithm 1.

In the following, we explain the differences between the water-filling power allocation (22) with Gaussian distribution inputs and the power allocation (37) with finite-alphabet inputs. To facilitate the presentation, let us introduce a function, $G_{2i-1}(\lambda)$, as follows

$$G_{2i-1}(\lambda) = \begin{cases} \frac{|H_{2i-1}|^2}{4\sigma^2 W \lambda} - \text{MMSE}_{2i-1}^{-1} \left(\frac{4\sigma^2 W \lambda}{|H_{2i-1}|^2} \right), & 0 < \lambda \leq \frac{|H_{2i-1}|^2}{4\sigma^2 W}; \\ 1, & \text{otherwise.} \end{cases} \quad (39)$$

Algorithm 1 Bisection Method

Input: Given $\lambda \in [0, \hat{\lambda}]$, $\delta > 0$, and initialize $\lambda_{\min} = 0$, $\lambda_{\max} = \hat{\lambda}$, where δ denotes termination parameter, and $\hat{\lambda}$ represents an upper bound of λ ;

1: **while** $\lambda_{\max} - \lambda_{\min} \geq \delta$ **do**

2: Set $\lambda = (1/2) (\lambda_{\min} + \lambda_{\max})$;

3: If $\lambda < |H_{2i-1}|^2 / 4\sigma^2 W$, substitute λ to obtain $p_{2i-1}^* = \frac{4\sigma^2 W}{|H_{2i-1}|^2} \text{MMSE}_{2i-1}^{-1} \left(\frac{4\sigma^2 W}{|H_{2i-1}|^2} \lambda \right)$; otherwise $p_{2i-1}^* = 0$;

4: If $\sum_{i=1}^{N/2} p_{2i-1}^* \leq \varepsilon$, set $\lambda_{\max} \leftarrow \lambda$; otherwise $\lambda_{\min} \leftarrow \lambda$;

5: **end while**

Output: p_{2i-1}^* ;

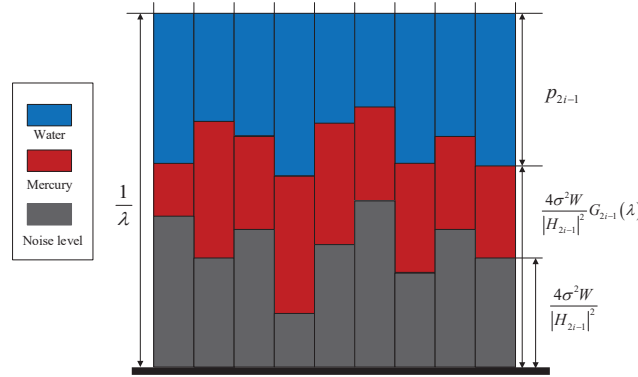


Fig. 2: Mercury water-filling scheme.

Then, the power allocation p_{2i-1} can be represented as

$$p_{2i-1} = \frac{1}{\lambda} - \frac{4\sigma^2 W}{|H_{2i-1}|^2} G_{2i-1}(\lambda). \quad (40)$$

Note that if $G_{2i-1}(\lambda) = 1$, the power allocation (40) is equivalent to the water-filling power allocation (22) with the Gaussian distribution inputs; otherwise, the power allocation (40) is the power allocation (37) with finite-alphabet inputs². Furthermore, based on the function $G_{2i-1}(\lambda)$, the power allocation (40) can be interpreted as the mercury-water-filling scheme [26]. More specifically, the power allocation (40) is illustrated in Fig. 2. For the $(2i - 1)$ th subcarrier with noise level $\frac{4\sigma^2 W}{|H_{2i-1}|^2}$, we first fill mercury to the height $\frac{4\sigma^2 W}{|H_{2i-1}|^2} G_{2i-1}(\lambda)$, then, pouring water

²The ACO-OFDM with finite-alphabet inputs converges weakly to that with Gaussian distribution inputs as the number of subcarriers N is sufficiently large [43].

(power) to the height $\frac{1}{\lambda}$. Note that both the noise level and the mercury in the mercury-water-filling scheme form the bottom level of the water-filling scheme.

C. Lower Bound of Mutual Information

For finite-alphabet inputs, the optimal power allocation scheme (37) involves the calculation of integrals of the MMSE ranged from $-\infty$ to $+\infty$, which can only be obtained by Monte Carlo and numerical integral methods at the expense of high computational complexity [21], [26]. To strike a balance between complexity and performance, we further develop a low complexity power allocation scheme.

As mentioned above, the expression of mutual information of ACO-OFDM is given in (24). The upper bound of the expectation term in (24) is given as [44]

$$\mathbb{E}_Z \left\{ \log_2 \sum_{k=1}^M \exp(-d_{nk}) \right\} \leq \log_2 \sum_{k=1}^M \mathbb{E}_Z \{ \exp(-d_{nk}) \} \quad (41a)$$

$$= \log_2 \sum_{k=1}^M \int_Z \exp(-d_{nk}) \frac{1}{\sqrt{2\pi W\sigma}} \exp\left(-\frac{|Z_{2i-1}|^2}{2\sigma^2 W}\right) dZ \quad (41b)$$

$$= \log_2 \sum_{k=1}^M \frac{1}{2} \exp\left(-\frac{\text{Re}^2\{C_{2i-1}\} + \text{Im}^2\{C_{2i-1}\}}{2\sigma^2 W}\right) \quad (41c)$$

$$= -1 + \log_2 \sum_{k=1}^M \exp\left(-\frac{p_{2i-1}|H_{2i-1}|^2|X_{2i-1,n} - X_{2i-1,k}|^2}{8\sigma^2 W}\right), \quad (41d)$$

where inequality (41a) is based on Jensen's inequality and (41b) is the expectation over Z . Note that the upper bound in (41d) is a deterministic value without involving integration which can be adopted in the following for the development of computationally efficient resource allocation algorithm. Let $R_L(\{p_{2i-1}\})$ represents the lower bound of mutual information in the $(2i-1)$ th subcarrier. Thus, the lower bound of achievable rate of the ACO-OFDM VLC system with finite-alphabet inputs is given by

$$\begin{aligned} R_L(\{p_{2i-1}\}) &= W \left(\log_2 M + 1 - \frac{1}{\ln 2} - \sum_{n=1}^M \frac{1}{M} \log_2 \sum_{k=1}^M \exp\left(-\frac{p_{2i-1}|H_{2i-1}|^2|X_{2i-1,n} - X_{2i-1,k}|^2}{8\sigma^2 W}\right) \right). \end{aligned} \quad (42)$$

Then, the SE of the lower bound of mutual information $\text{SE}_L(\{p_{2i-1}\})$ can be expressed as

$$\text{SE}_L(\{p_{2i-1}\}) = \frac{\sum_{i=1}^{N/2} R_L(\{p_{2i-1}\})}{2NW}. \quad (43)$$

By substituting the derived lower bound (42) into the objective of problem of (43), the SE maximization problem (15) can be reformulated as follows

$$\underset{\{p_{2i-1}\}}{\text{maximize}} \quad \text{SE}_L(\{p_{2i-1}\}) \quad (44a)$$

$$\text{s.t.} \quad \sum_{i=1}^{N/2} p_{2i-1} \leq \varepsilon, \quad (44b)$$

$$p_{2i-1} \geq 0, \quad i = 1, \dots, N/2, \quad (44c)$$

where $\varepsilon \triangleq \min \left\{ P, \frac{4P_o^2}{\mathbb{E}^2\{|X_{2i-1}|\}} \right\}$. This optimization problem (44) is a standard convex problem, which can be efficiently solved by the interior-point algorithm [38], [45].

IV. ENERGY EFFICIENCY OF ACO-OFDM

In this section, we investigate the design of optimal power allocation scheme to maximize the EE of the ACO-OFDM VLC system which is subject to the QoS requirement, and both the electrical and optical power constraints. Mathematically, we can formulate the EE maximization problem of the ACO-OFDM VLC system as

$$\underset{\{p_{2i-1}\}}{\text{maximize}} \quad \frac{\sum_{i=1}^{N/2} R_{2i-1}(\{p_{2i-1}\})}{\sum_{i=1}^{N/2} p_{2i-1} + P_c} \quad (45a)$$

$$\text{s.t.} \quad \mathbb{E}\{\hat{x}_k\} \leq P_o, \quad (45b)$$

$$\sum_{i=1}^{N/2} p_{2i-1} \leq P, \quad (45c)$$

$$\sum_{i=1}^{N/2} R_{2i-1}(\{p_{2i-1}\}) \geq r, \quad (45d)$$

$$p_{2i-1} \geq 0, \quad i = 1, \dots, N/2, \quad (45e)$$

where r is the minimum achievable rate requirement.

Problem (45) is a general formulation of the EE in ACO-OFDM VLC systems. In the following, we will present the optimal EE scheme of problem (45) with Gaussian distribution inputs and finite-alphabet inputs respectively. Note that both objective function (45a) and the constraint (45d) of the EE maximization problem (45) are different from that of the SE maximization problem (15).

A. Gaussian Distribution Inputs

With Gaussian distribution inputs and the achievable rate expression (18), the EE of Gaussian distribution $EE_G(\{p_{2i-1}\})$ can be expressed as

$$EE_G(\{p_{2i-1}\}) = \frac{\sum_{i=1}^{N/2} W \log_2 \left(1 + \frac{p_{2i-1} |H_{2i-1}|^2}{4\sigma^2 W} \right)}{\sum_{i=1}^{N/2} p_{2i-1} + P_c}. \quad (46)$$

The EE maximization problem with Gaussian distribution inputs of ACO-OFDM systems can be formulated as

$$\underset{\{p_{2i-1}\}}{\text{maximize}} \quad EE_G(\{p_{2i-1}\}) \quad (47a)$$

$$\text{s.t.} \quad \sum_{i=1}^{N/2} p_{2i-1} \leq \min \{P, 2N\pi P_o^2\}, \quad (47b)$$

$$\sum_{i=1}^{N/2} W \log_2 \left(1 + \frac{p_{2i-1} |H_{2i-1}|^2}{4\sigma^2 W} \right) \geq r, \quad (47c)$$

$$p_{2i-1} \geq 0, \quad i = 1, \dots, N/2. \quad (47d)$$

For the objective function (47a), the numerator is a differentiable concave function of the variable p_{2i-1} , while the denominator is an affine function of p_{2i-1} . Thus, the objective function (47a) is a quasi-concave function of p_{2i-1} . With a convex constraint set, problem (47) is a typical fractional problem [46], which is generally non-convex. To circumvent the non-convexity, Dinkelbach-type iterative algorithm [47]–[49] can be adopted to tackle the problem by converting the problem (47) into a sequence of convex subproblems. In particular, solving these convex subproblems iteratively can eventually obtain the globally optimal solution of problem (47).

Specifically, let Υ denote the feasible set defined constraints of problem (47) as follows

$$\Upsilon = \{p_{2i-1} | \underline{P} \leq p_{2i-1} \leq \overline{P}\}, \quad i = 1, \dots, N/2, \quad (48)$$

where $\underline{P} \triangleq \frac{(2^{r/W}-1)4\sigma^2W}{|H_{2i-1}|^2}$ and $\overline{P} \triangleq \min\{P, 2N\pi P_o^2\}$.

Moreover, we introduce a new function $f(\{p_{2i-1}\})$ as follows

$$f(\{p_{2i-1}\}) \triangleq \sum_{i=1}^{N/2} W \log_2 \left(1 + \frac{p_{2i-1}|H_{2i-1}|^2}{4\sigma^2W} \right) - q \left(\sum_{i=1}^{N/2} p_{2i-1} + P_c \right), \quad (49)$$

where q is a given non-negative parameter to be found iteratively. Then, by calculating the roots of the equation $f(\{p_{2i-1}\}) = 0$ in the set Υ , the optimal solution of problem (47) can be obtained.

For a given q in each iteration, the convex subproblem over p_{2i-1} can be expressed as

$$\underset{\{p_{2i-1}\}}{\text{maximize}} \quad f(\{p_{2i-1}\}) \quad (50a)$$

$$\text{s.t. } p_{2i-1} \in \Upsilon. \quad (50b)$$

Since the transformed problem is convex and satisfies the Slater's constraint qualification, we apply the conventional optimization techniques by taking the partial derivative of function $f(\{p_{2i-1}\})$ and setting it to zero, i.e. $\frac{\partial f(\{p_{2i-1}\})}{\partial p_{2i-1}} = 0$, which yields

$$\tilde{p}_{2i-1} = \left[\frac{W}{q \ln 2} - \frac{4\sigma^2W}{|H_{2i-1}|^2} \right]^+. \quad (51)$$

Then, by projecting \tilde{p}_{2i-1} into the feasible region Υ , we obtain the optimal power p_{2i-1}^* of problem (50) as follows

$$p_{2i-1}^* = \text{Proj}_{\Upsilon}(\tilde{p}_{2i-1}) = \begin{cases} \underline{P}, & \tilde{p}_{2i-1} < \underline{P}; \\ \tilde{p}_{2i-1}, & \underline{P} \leq \tilde{p}_{2i-1} \leq \overline{P}; \\ \overline{P}, & \tilde{p}_{2i-1} \geq \overline{P}, \end{cases} \quad (52)$$

where $\text{Proj}_{\Upsilon}(\tilde{p}_{2i-1})$ denotes the projection of \tilde{p}_{2i-1} into the subspace Υ .

Note that if $\tilde{p}_{2i-1} \in \Upsilon$, the power allocation (52) for the EE maximization is a generalization of the power allocation (22) for the SE maximization. In particular, for a small electrical transmit power budget, both SE and EE maximization show the same water-filling solution. However, when the budget is sufficiently large, once the maximum EE is achieved, the optimal EE power

algorithm would clip the transmit power level, as can be observed in (52).

Finally, the EE maximization problem of Gaussian distribution inputs can be solved by the Dinkelbach-type algorithm. Under a finite number of iterations, the Dinkelbach-type algorithm is guaranteed to converge to the optimal solution of problem (47), e.g. [47]–[49]. Algorithm 2 shows the detail of implementation.

Algorithm 2 Dinkelbach-type Algorithm

Input: Given $\delta \rightarrow 0, n = 0, p_{2i-1}^* > 0, q^{(n)} = 0$;
1: **while** $|q^{(n)} - q^{(n+1)}| \leq \delta$ **do**
2: Compute the optimal solution p_{2i-1}^* ;
3: Calculating the value of function $f(\{p_{2i-1}^*\})$;
4: $q^{(n+1)} = \text{EE}(p_{2i-1}^*)$;
5: $n = n + 1$;
6: **end while**
Output: $\text{EE}(p_{2i-1}^*)$;

B. Finite-alphabet Inputs

For the finite-alphabet inputs, the achievable rate expression is given by (23), thus, the EE of finite-alphabet inputs $\text{EE}_F(\{p_{2i-1}\})$ is given by

$$\text{EE}_F(\{p_{2i-1}\}) = \frac{\sum_{i=1}^{N/2} W \left(\log_2 M - \frac{1}{\ln 2} - \sum_{n=1}^M \frac{1}{M} \mathbb{E}_Z \left\{ \log_2 \sum_{k=1}^M \exp \left(- \frac{|\frac{1}{2} H_{2i-1} \sqrt{p_{2i-1}} (X_{2i-1,n} - X_{2i-1,k}) + Z_{2i-1}|^2 \right)}{\sigma^2 W} \right\} \right)}{\sum_{i=1}^{N/2} p_{2i-1} + P_c}. \quad (53)$$

Furthermore, the average optical power constraint (45b) can be restricted to constraint (27). Thus, the optimal EE maximization problem with finite-alphabet inputs under the electrical power constraint, the average optical power constraint, and the minimum rate constraint can be

expressed as

$$\underset{\{p_{2i-1}\}}{\text{maximize}} \quad \text{EE}_F(\{p_{2i-1}\}) \quad (54a)$$

$$\text{s.t.} \quad \sum_{i=1}^{N/2} p_{2i-1} \leq \min \left\{ P, \frac{4P_o^2}{\mathbb{E}^2\{|X_{2i-1}|\}} \right\}, \quad (54b)$$

$$\sum_{i=1}^{N/2} R_F(\{p_{2i-1}\}) \geq r, \quad (54c)$$

$$p_{2i-1} \geq 0, \quad i = 1, \dots, N/2. \quad (54d)$$

Note that there is no closed-form expression for the achievable rate in (54a), although $R_F(\{p_{2i-1}\})$ is strictly concave over its input power. On the other hand, constraints (54b)-(54d) form a convex feasible solution set. Thus, problem (54) is a concave-linear fractional problem that can be solved by Dinkelbach-type algorithms. The details are omitted as it is similar to the case adopting Gaussian distribution inputs as discussed in Section IV-A.

C. Lower Bound of Mutual Information

Note that in the objective function (54a), complicated integrals need to be solved with high computational complexity. To reduce the complexity, we exploit the lower bound of achievable rate (42), and the corresponding EE function of the ACO-OFDM VLC system is given by

$$\begin{aligned} & \text{EE}_L(\{p_{2i-1}\}) \\ &= \frac{\frac{NW}{2} \left(\log_2 M + 1 - \frac{1}{\ln 2} \right) - \sum_{i=1}^{N/2} \sum_{n=1}^M \frac{W}{M} \log_2 \sum_{k=1}^M \exp \left(-\frac{\left| \frac{1}{2} H_{2i-1} \sqrt{p_{2i-1}} (X_{2i-1,n} - X_{2i-1,k}) \right|^2}{2\sigma^2 W} \right)}{\sum_{i=1}^{N/2} p_{2i-1} + P_c}. \end{aligned} \quad (55)$$

Furthermore, the EE problem (45) can be reformulated as

$$\underset{\{p_{2i-1}\}}{\text{maximize}} \quad \text{EE}_L(\{p_{2i-1}\}) \quad (56a)$$

$$\text{s.t.} \quad \sum_{i=1}^{N/2} p_{2i-1} \leq \min \left\{ P, \frac{4P_o^2}{\mathbb{E}^2\{|X_{2i-1}|\}} \right\}, \quad (56b)$$

$$\sum_{i=1}^{N/2} R_L(\{p_{2i-1}\}) \geq r, \quad (56c)$$

$$p_{2i-1} \geq 0, \quad i = 1, \dots, N/2. \quad (56d)$$

As the lower bound of the achievable rate (42) is concave and differentiable, problem (56) is also a concave-linear fractional problem. At the same time, problem (56) can also be solved by using Dinkelbach-type algorithms.

V. SIMULATION RESULTS AND DISCUSSION

This section presents numerical results to evaluate the proposed power allocation schemes for SE maximization and EE maximization problems of ACO-OFDM VLC systems. Consider an indoor ACO-OFDM VLC system installed with four LEDs, where a corner of a square room denotes the origin $(0, 0, 0)$ of a three-dimensional Cartesian coordinate system (X, Y, Z) . The location of receiver is $(0.5, 1, 0)\text{m}$, and the locations of four LEDs are $(1.5, 1.5, 3)\text{m}$, $(1.5, 3.5, 3)\text{m}$, $(3.5, 1.5, 3)\text{m}$, and $(3.5, 3.5, 3)\text{m}$, respectively. The other basic parameters of the VLC system are listed in Table I.

A. Simulation Results of SE Maximization Problem

In this subsection, we present the results of the proposed three power allocation schemes for maximizing the SE for Gaussian distribution inputs, finite-alphabet inputs, and lower bound of the mutual information.

TABLE I: Simulation Parameters of the ACO-OFDM VLC System.

Definition	Value
Number of subcarriers, N	64
Transmit angle, θ	60°
FOV, Ψ	90°
Lambertian emission order, m	1
Half power angle, $\Phi_{1/2}$	60°
PD collection area, A_r	1 cm^2
Circuit power consumption, P_c	0.1 W
Angle of arrival/departure, φ	45°
Optical filter gain of receiver, $T(\varphi)$	0 dB
Concentrator gain of receiver, $G(\varphi)$	0 dB
Noise PSD, σ^2	$10^{-18} \text{ A}^2/\text{Hz}$
Modulation order, M	4-QAM
Bandwidth of each subcarrier, W	1 MHz

Fig. 3 illustrates allocated power p_i versus channel gain H_i of subcarrier i of SE_G , SE_F , and SE_L , where $P = 20$ (W), $P_o = 0.25$ (W). As can be observed from Fig. 3, the value of the allocated power p_i of the SE_G is proportional to the corresponding noise level $\frac{4\sigma^2 W}{|H_{2i-1}|^2}$, which is

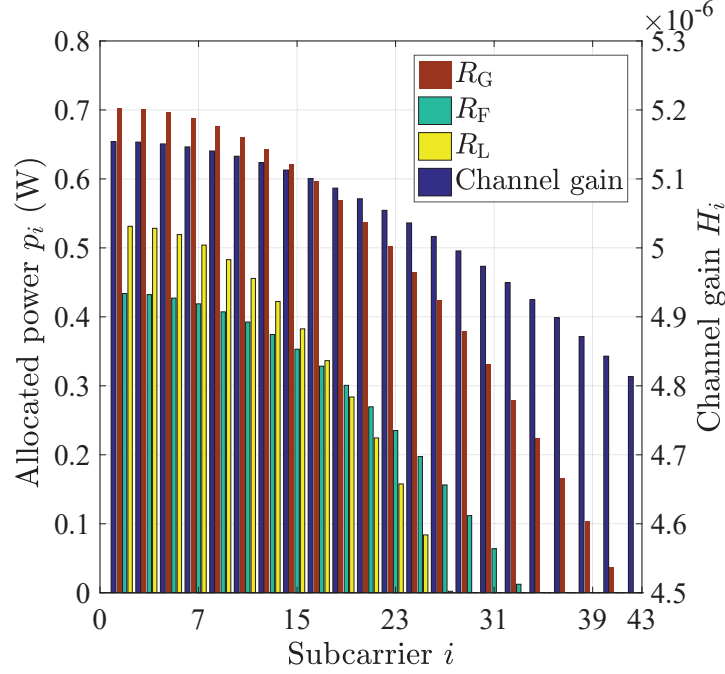


Fig. 3: Allocated power p_i versus channel gain H_i of subcarrier i of SE_G , SE_F , and SE_L with $P = 20$ W, $P_o = 0.25$ W.

due to the water-filling solution for the maximization of the SE with Gaussian distribution inputs. While for the SE_F case, the allocated power of p_i not only depends on the noise level, but also depends on the mercury level $\frac{4\sigma^2 W}{|H_{2i-1}|^2} G_{2i-1}(\lambda)$, which is due to the mercury-water-filling method for the maximization of the SE with finite-alphabet inputs. For the case of SE_L , the allocated power of each subcarrier is based on $\sum_{n=1}^M \log_2 \sum_{k=1}^M \exp\left(-\frac{p_{2i-1}|H_{2i-1}|^2 |X_{2i-1,n} - X_{2i-1,k}|^2}{8\sigma^2 W}\right)$. Since $2N\pi \geq \frac{4}{\mathbb{E}^2\{|X_{2i-1}|\}}$, we have $2N\pi P_o^2 \geq \frac{4P_o^2}{\mathbb{E}^2\{|X_{2i-1}|\}}$. Therefore, when $P \geq 2N\pi P_o^2$, the allocated power for Gaussian distribution inputs is more than that for finite-alphabet inputs. Moreover, due to the negligible small channel gain, the allocated power from the 43-th subcarrier to the 63-th subcarrier of the three cases are zero, which are not shown in Fig. 3 for brevity.

Fig. 4(a) illustrates SE_G , SE_F , and SE_L versus the electrical power threshold P with an optical power constraint $P_o = 0.25$ (W) and $P_o = \infty$ (without optical power constraint), respectively. As shown in Fig. 4(a), for $P_o = \infty$ case, as P increases, SE_G keeps increasing, while SE_F and SE_L first increase and then remain constant. Moreover, when P is large, SE_G is higher than both SE_F and SE_L . This is because the Gaussian distribution can be regarded as a very high-order constellation modulation input, which is more suitable for high SNRs. While for $P_o = 0.25$ (W) case, as P increases, SE_G , SE_F , and SE_L first increase and then remain constant. This is because

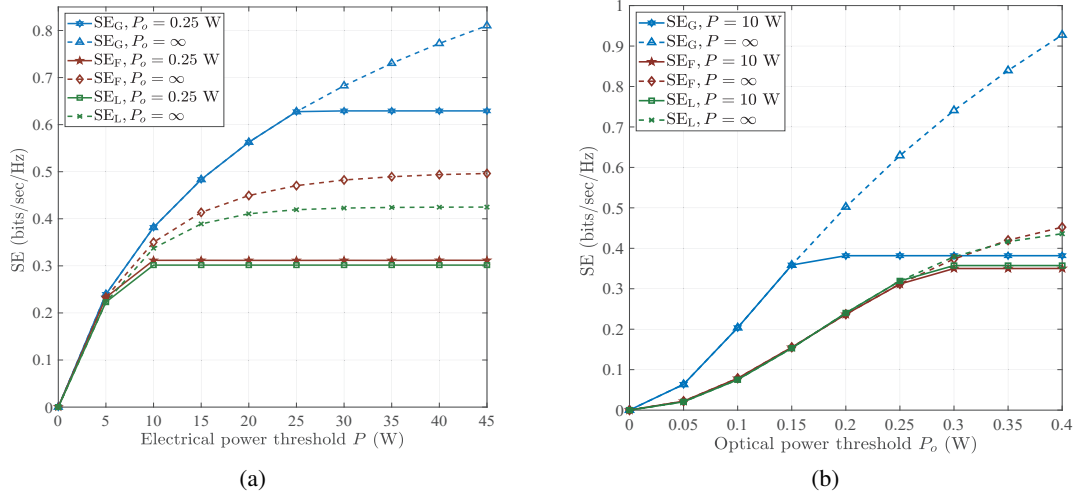


Fig. 4: (a) SE_G , SE_F , and SE_L versus electrical power threshold P ; (b) SE_G , SE_F , and SE_L versus optical power threshold P_o .

the allocated power is limited by the optical power threshold, i.e., $P_o = 0.25$ (W). Moreover, it can be observed from Fig. 4(a) that the SE_F with finite-alphabet inputs reaches the maximum point faster than the SE_G with Gaussian distribution inputs. The reason is that the coefficient of P_o of Gaussian distribution inputs $2N\pi$ in (20b) is larger than that of finite-alphabet inputs $\frac{4}{\mathbb{E}^2\{|X_{2i-1}|\}}$ in (29b). Besides, the gap between SE_F and SE_L is small, which means SE_L could approximate SE_F well enough.

Fig. 4(b) illustrates SE_G , SE_F , and SE_L versus optical power threshold P_o with an electrical power constraint $P = 10$ (W) and $P = \infty$ (without electrical power constraint), respectively. It can be seen in Fig. 4(b), for $P = \infty$ case, as P_o increases, SE_G keep increasing, while SE_F and SE_L first increase and then remain constant. Besides, when P_o is large, SE_G is higher than both SE_F and SE_L . The reason is that the Gaussian distribution is more suitable for high SNRs, which is similar as that in Fig. 4(a). While for $P = 10$ (W) case, SE_G , SE_F , and SE_L first increase and remain as a constant. In fact, the allocated power is restricted by the electrical power constraint, i.e., $P = 10$ (W). When P_o is small, SE_G is higher than SE_F and SE_L . As P_o increases, SE_G first reaches its maximum point, while SE_F and SE_L reach to the corresponding saturation points later. The reason is that the coefficient of P_o of Gaussian distribution inputs $2N\pi$ in (20b) is larger than that of finite-alphabet inputs $\frac{4}{\mathbb{E}^2\{|X_{2i-1}|\}}$ in (29b). Thus, with the same optical power threshold P_o , the Gaussian distribution inputs can allocate more power than that of finite-alphabet inputs, which was also verified in Fig. 3.

B. Simulation Results of EE Maximization Problems

In this subsection, we present the simulation results for the evaluation of the EE performance of Gaussian distribution inputs, finite-alphabet inputs case, and lower bound of mutual information case for ACO-OFDM VLC systems.

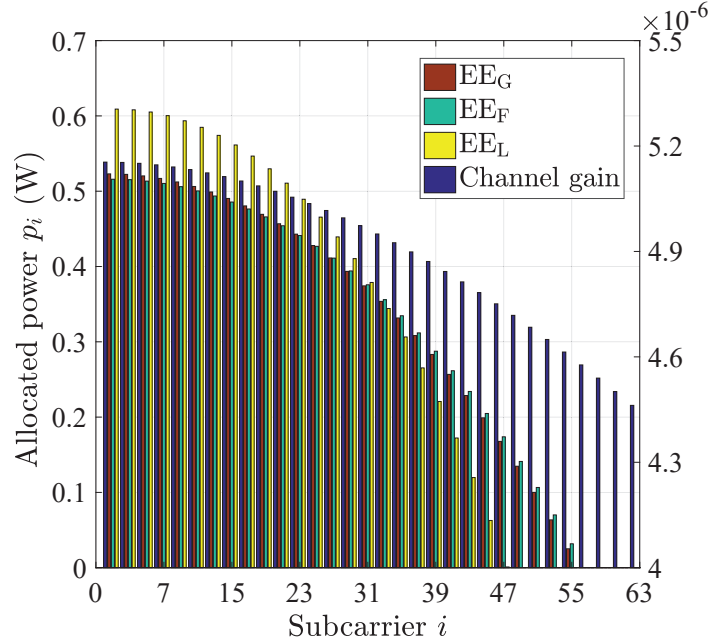


Fig. 5: Allocated power p_i versus channel gain H_i of subcarrier i of EE_G , EE_F , and EE_L with $P = 20$ W, $P_o = 0.25$ W.

Fig. 5 illustrates the different allocated power p_i of EE_G , EE_F , and EE_L versus channel gain H_i of subcarrier i respectively, where $P = 20$ (W), $P_o = 0.25$ (W), and $r = 1$ (bits/sec/Hz). From Fig. 5, we can see that the allocated power of subcarrier i of EE_G is proportional to its channel gain, which is due to the power allocation strategy in (51) and (52). While the allocated power of subcarrier i of both EE_F and EE_L depend on both channel gains and MMSE functions. Compared with SE maximization problems, both the objective function and rate constraints of the EE maximization problems are different, which accounts for the different power allocation in Fig. 5 compared with that in Fig. 3. In particular, the proposed algorithm avoids allocating exceedingly large amount of powers to subcarriers to maximize the system EE.

Fig. 6(a) depicts EE_G , EE_F , and EE_L versus electrical power threshold P with an optical power threshold $P_o = 0.03$ (W) and $P_o = \infty$ (without optical power constraint) respectively, where the rate constraint $r = 0.1$ (bits/sec/Hz). We see from Fig. 6(a) that for $P_o = \infty$ case, as P increases, EE_G , EE_F , and EE_L first increase and then remain constant. This is because the

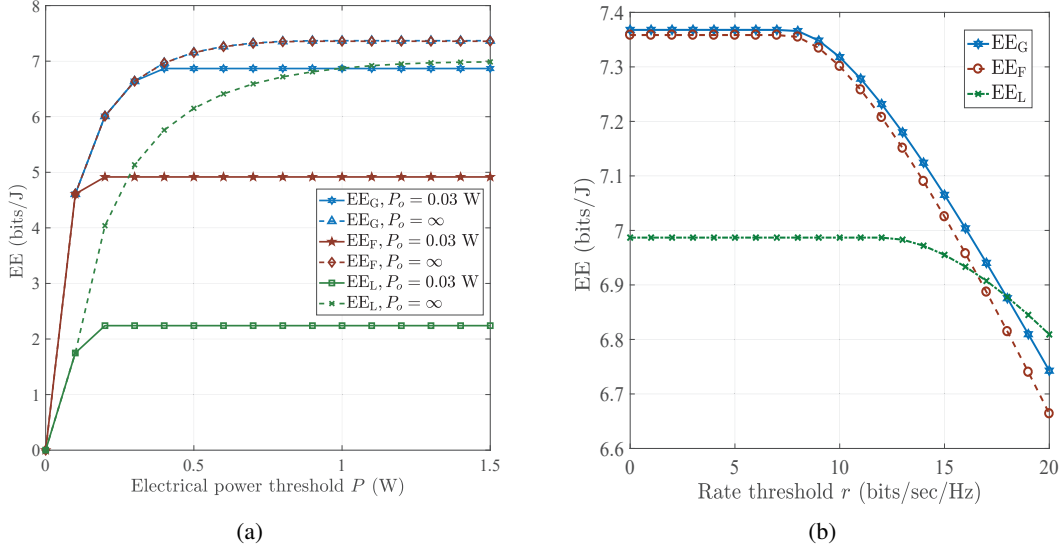


Fig. 6: (a) EE_G , EE_F , and EE_L versus electrical power threshold P with rate constraint $r = 0.1$ bits/sec/Hz and two different optical power thresholds $P_o = 0.03$ W, and $P_o = \infty$; (b) EE_G , EE_F , and EE_L versus rate threshold r with power threshold $P = 20$ W and optical power threshold $P_o = 1$ W.

optimal EE remains a constant when it has reached the maximum value. Moreover, the value of EE_F approaches to that of EE_G , which are higher than the value of EE_L . While for $P_o = 0.03$ case, as P increases, the EE_G , EE_F , and EE_L first increase and then remain constant. The reason is that EE_G , EE_F , and EE_L is limited by the optical power constraint $P_o = 0.03$ (W). Besides, for the large P , the value of EE_G are the highest of the three power allocation schemes, while EE_L is the lowest.

Fig. 6(b) depicts EE_G , EE_F , and EE_L versus rate threshold r with power threshold $P = 20$ (W) and optical power threshold $P_o = 1$ (W). As shown in Fig. 6(b), EE_G is higher than EE_F and EE_L , and the gap between EE_G and EE_F is small. Moreover, the EE of three cases first remains constant and then decreases, as the rate threshold r increases. Indeed, when the value of the rate threshold r is small, the performed power allocation can easier in satisfying the rate requirement and thus the EE does not change. While for a high rate threshold r , the resource allocation in the system becomes less feasible in allocating power as it is forced to consume more power to satisfy the stringent rate constraint, and therefore the optimal EE decreases. Moreover, the gap between EE_F and EE_L increases as rate threshold r increases.

C. Relationship Between SE and EE

To guarantee the QoS to users with affordable energy, EE and SE in a specific system are used to evaluate the performances of energy and spectral usage. Especially, for achieving a good balance performance of VLC equipments, the tradeoff between SE and EE should be delicately considered. Based on (19) and (46), the relationship between SE and EE of Gaussian distribution for ACO-OFDM can be discussed as

$$EE_G(\{p_{2i-1}\}) = \frac{2NW}{\sum_{i=1}^{N/2} p_{2i-1} + P_c} SE_G(\{p_{2i-1}\}). \quad (57)$$

Similarly, the relationship between EE and SE of finite-alphabet inputs and the lower bound of mutual information for ACO-OFDM are similar to that of the Gaussian distribution.

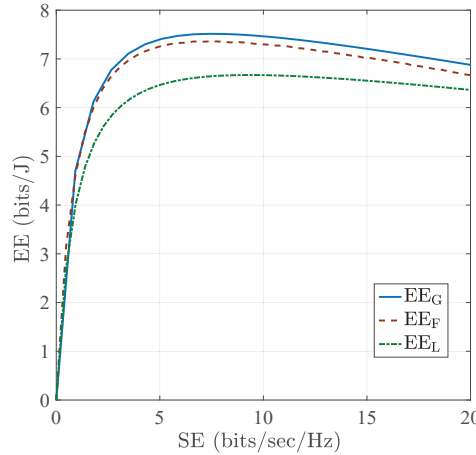


Fig. 7: EE_G , EE_F , and EE_L versus the SE_G , SE_F , and SE_L with optical power threshold $P_o = 0.03$ W.

Fig. 7 shows the EE_G , EE_F , and EE_L versus the SE_G , SE_F and SE_L with optical power threshold $P_o = 0.03$ (W). It can be seen from Fig. 7 that there is a non-trivial tradeoff between the system SE and EE. In practice, as SE increases, the EE increases at first and then decreases. In particular, there exists an optimal SE to maximize EE. Moreover, the peak of EE_G is the highest while the maximum EE_F is higher than that of EE_L , and this phenomenon was also verified in Fig. 6 (a) and (b).

VI. CONCLUSION

In this study, we addressed the problem of designing optimal power allocation schemes to maximize the SE and the EE of ACO-OFDM in VLC systems with Gaussian distributions

and finite-alphabet inputs. We first derived the achievable rates and the average optical power constraint for ACO-OFDM VLC systems with the above two mentioned inputs. Then, we derived the optimal power allocation schemes to maximize the SE of ACO-OFDM systems. Specifically, for Gaussian distribution inputs, the water-filling-based power allocation scheme was presented to maximize the SE. By exploiting the relationship between the mutual information and MMSE, the optimal power allocation scheme was derived to maximum the SE system with finite-alphabet inputs. We further developed the optimal power allocation scheme to maximize the EE of ACO-OFDM VLC systems with Gaussian distributions and finite-alphabet inputs. By adopting Dinkelbach-type algorithm, the EE maximization problems were transformed into convex problems and the interior point algorithm was exploited to obtain the optimal solution. Besides, to reduce the computational complexity of finite-alphabet inputs cases, we derived the closed-form lower bounds of mutual information for both the SE and the EE maximization problems. Finally, we showed the relationship between the SE and the EE of ACO-OFDM VLC systems.

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