Description of the IMACLIM-Country model: A country-scale computable general equilibrium model to assess macroeconomic impacts of climate policies

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A country-scale computable general equilibrium model to assess macroeconomic impacts of climate policies

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1 Imaclim-Country notations

Calibration consists in providing a set of values to all variables and then determining the values that should be given to the parameters so that the set of equations defining the model holds. The exercise is therefore to determine what values the parameters must take in order for the values drawn from national accounts to be linked by the set of equations. However, all parameters do not receive their values from the calibration: the carbon tax, for instance, is a purely exogenous parameter; other parameters have their values set according to some econometric estimation on data superseding the national accounts as described by the input-output table and the economic account table. As a result of these distinctions, the notations below are presented in three categories, (i) the variables of the model properly speaking, (ii) the parameters of the model that are calibrated on statistical data, and (iii) the exogenous parameters. Within each of these categories the notation are listed in alphabetical order (the Greek letters are classified according to their English name rather than according to their equivalent in the Latin alphabet).

1.1 Variables ofImaclim-Country

Variable Name	**Description**
α_{ij}	Technical coefficient, quantity of good i entering the production of one good j
OT	Other transfers
OT_H	Other transfers to the households
OT_F	Other transfers to firms
OT_G	Other transfers to the public administrations
AFC_H	Self-financing capacity of class h
AFC_F	Self-financing capacity of firms
AFC_G	Self-financing capacity of the public administrations
AFC_{ROW}	Self-financing capacity of the rest of the world
C_{i_h}	Final consumption of good i by household class h
D_h	Net debt of class h
D_F	Net debt of firms
D_G	Net public debt
D_{ROW}	Net debt of the rest of the world
d_i	Reform-induced interest rate differential
GOS_H	Gross operating surplus accruing to households
GOS_F	Gross operating surplus accruing to firms
GOS_G	Gross operating surplus accruing to public administrations
$GFCF_h$	Gross fixed capital formation of household class h

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$GFCF_F$	Gross fixed capital formation of firms
$GFCF_G$	Gross fixed capital formation of public administrations
$\gamma_{IC_{ij}}$	CO_2 emissions per unit of good i consumed in the production of good j
γ_{FC_i}	CO_2 emissions per unit of good i consumed by households
G_i	Final public consumption of good i
i_H	Effective interest rate on the net debt of households
i_F	Effective interest rate on the net debt of firms
i_G	Effective interest rate on the net debt of public administrations
I_i	Final consumption of good i for the investment
CPI	Consumer price index (Fisher)
k_i	Capital intensity of good i
l_i	Labour intensity of good i
LS_H	Lump-sum transfers from carbon tax revenues to households
LS_F	Lump-sum transfers from carbon tax revenues to firms
M_i	Imports of good i
SM	Sum across goods and uses of the specific sale margins
N	Total population
NL	Total active population (full time equivalent)
p_{M_i}	Import price of good i
p_i	Average price of the resource in good i (domestically produced and imported)
$p_{IC_{ij}}$	Price of good i for the production of good j
p_{C_i}	Consumption price of good i
p_{G_i}	Public price of good i
p_{I_i}	Investment price of good i
Φ_i	Endogenous technical progress coefficient applying to the production of good i
p_K	Cost of capital input (weighted sum of investment prices)
p_{L_i}	Cost of labour input in the production of good i
p_{X_i}	Export price of good i
p_{Y_i}	Production price of good i
RBT_F	Before-tax gross disposable income of firms
RBT_h	Before-tax gross disposable income of household class h
RBT_H	Before-tax gross disposable income of all households classes (m)
R_F	Gross disposable income of firms
R_G	Gross disposable income of public administrations
R_h	Gross disposable income of household class h
R_{EXP_h}	Consumed income of household class h
R_{OSB}	Sum of social transfers to households not elsewhere included
R_U	Sum of unemployment benefits
R_P	Sum of retirement pensions
σ_{Θ_i}	Elasticity of the decreasing returns coefficient of production i to its output.

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T	Total taxes and social contributions
T_L	Sum of social contributions of the employer and the employee
T_{En}	Fiscal revenues from excise tax on energy products
T_{OP}	Fiscal revenues of excise taxes other than the energy product tax
T_{VA}	Value-added tax revenues
T_F	Firms tax revenues
T_{I_h}	Revenue from household class h income tax payments
T_{D_h}	Revenue from other direct taxes paid by household class h
T_{carb}	Carbon tax revenues
Θ_i	Decreasing returns coefficient for the production of good i
τ_{LT}	Social contribution rate applicable to net wages
$\tau_{CM_{TRADE}}$	Trade mark-up on the commercial good or on the aggregate encompassing it
$\tau_{CM_{TRANS}}$	Transport mark-up on the transport good or on the aggregate encompassing it
u	Unemployment rate
u_h	Household class h unemployment rate
ω_i	Net wage in the production of good i
Ω	Average net wage across productions
X_i	Good i exports
Y_i	Good i production

Table 1 – Variables for solving IMACLIM ARGENTINA

1.2 Parameters calibrated on statistical data

Variable Name	**Description**
$\overline{L}$	Total active population in full-time equivalents
$\overline{L_h}$	Active population of household class h in full-time equivalents
λ_{ij}, λ_{L_i}, λ_{K_i}	Coefficients of the Constant Elasticity of Substitution (CES) production function governing the variables shares of conditional factor demands.
$\overline{N_h}$	Total population of household class h.
$\overline{N_{P_h}}$	Number of retirees of household class h.
$\overline{\omega_{OT_h}}$	Share of the other transfers accruing to households devoted to household class h.
$\overline{\omega_{OT_H}}$	Share of other transfers accruing to households (all classes together). Calibrated on the economic account table.
$\overline{\omega_{OT_F}}$	Share of other transfers accruing to firms. Calibrated on the economic account table (aggregate of financial and non financial firms, and of non-profit organisations).

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$\overline{\omega_{OT_G}}$	Share of other transfers accruing to public administrations. Calibrated on the economic account table.
$\overline{\omega_{K_h}}$	Share of the capital income of households accruing to household class *h*. Calibrated as the share accruing to household class *h* of revenues other than those of labour, in the m-class aggregation .
$\overline{\omega_{K_H}}$	Share of capital income accruing to households (all classes). Calibrated on the economic account table.
$\overline{\omega_{K_F}}$	Share of capital income accruing to firms. Calibrated on the economic account table (aggregate of financial and non financial firms, and of non-profit organisations).
$\overline{\omega_{K_G}}$	Share of capital income accruing to public administrations. Calibrated on the economic account table
$\overline{\pi_i}$	Mark-up rate of profit margins(rate of net operating surplus) in the production of good *i*. Calibrated as the ratio of net operating surplus to distributed output.
$\overline{t_{OPT_i}}$	Excise taxes other than the energy product tax per unit of consumption of good *i*. Calibrated as the ratio of the corresponding fiscal revenue of each good *i* (input-output table data after subtraction of the energy product tax) to total domestic consumption in the no-policy equilibrium $Yi0 + Mi0 - Xi0$ (exports are assumed to be exempted).
$\overline{t_{EnT_{FC_i}}}$	energy product tax per *toe* of energy product consumptions by households. The energy product tax is isolated from other excise taxes.
$\overline{t_{EnT_{IC_i}}}$	energy product tax per *toe* of energy product in intermediate consumptions. The energy product tax is isolated from other excise taxes.
$\overline{\tau_{TI_h}}$	Effective income tax rate of household class *h*. Calibrated as the ratio of income tax payments to the before-tax gross disposable income. Both aggregates are distributed among household classes based on the shares observed in the h-class aggregation .
$\overline{\tau_{T_F}}$	Effective firms tax rate. Calibrated as the ratio of the firms tax fiscal revenue, to the share of the gross operating surplus (GOS) accruing to firms.
$\overline{\tau_{SM_{IC_{ij}}}}$	Specific mark-up rate on intermediate consumptions (if *i* is not a hybrid good then the rate is nil). Defined during the hybridisation procedure.
$\overline{\tau_{SM_{C_i}}}$	Specific mark-up rate on household consumptions (if *i* is not a hybrid good then the rate is nil). Defined during the hybridisation procedure.
$\overline{\tau_{SM_{G_i}}}$	Specific mark-up rate on public consumptions (if *i* is not a hybrid good then the rate is nil). Defined during the hybridisation procedure. Under the convention that public energy consumptions are nil.
$\overline{\tau_{SM_{I_i}}}$	Specific mark-up rate on investment (if *i* is not a hybrid good then the rate is nil). Defined during the hybridisation procedure.
$\overline{\tau_{SM_{X_i}}}$	Specific mark-up rate on exports (if *i* is not a hybrid good then the rate is nil). Defined during the hybridisation procedure.

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$\overline{\tau_{S_h}}$	Savings rate of household class h. Calibrated as the ratio of the savings of class h to its gross disposable income (R_h), with the data being derived from all the main data sources.
$\overline{\tau_{VAT_i}}$	Value-added tax rate applying to the final consumption of good i. Calibrated on input-output table data by treating the VAT as a simple sales tax levied indifferently on C, G and i.

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Table 2 – Calibrated parameters for Imaclim-Country

1.3 Exogenous parameters

Variable Name	**Description**
$\beta_{\omega_{CPI}}$	Indexation coefficient of wage on consumer price in the wage curve. The value "0" stands for no-indexation of wages on consumer price - wage curve is on nominal wage. The value "1" stands for a complete indexation of the wage curve on consumer price.
β_{i_h}	Share of the good i consumption of household class h that corresponds to a basic need. Set for each good i at a level that defines a basic need equal to 80% of the real consumption of the class for which it is the lowest.
$\beta_{IC_{ji}}$	Technical asymptote of the technical coefficient α_{ij} .
β_{K_i}	Technical asymptote of the capital intensity of good i.
β_{L_i}	Technical asymptote of the labour intensity of good i.
δ_{M_i}	Evolution rate of the import of good i excluding price effect.
δ_{X_i}	Evolution rate of the export of good i excluding price effect.
$\delta_{p_{M_i}}$	Evolution rate of the p_{M_i} price of imported good j.
$\delta_{p_{X_i}}$	Evolution rate of the p_{X_i} price of exported good j.
ΦL_i	Labour productivity improvements of good i.
σ_i	Substitution elasticity of the variable shares of production factors.
σ_{CR_i}	Income-elasticity of household consumption of good i.
σ_{CP_i}	Price-elasticity of household consumption of good i.
$\sigma_{M_{p_i}}$	Elasticity of the ratio of imports to domestic production of good i, to the corresponding terms of trade.
$\sigma_{X_{p_i}}$	Elasticity of good i exports to the corresponding terms of trade.
σ_{w_u}	Elasticity of the average net wage (nominal or real, see supra) to the unemployment rate.
$t_{carb_{IC}}$	Carbon tax on the carbon emissions of intermediate consumptions.
$t_{carb_{FC}}$	Carbon tax on the carbon emissions of household consumptions.

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Table 3 – Exogenous parameters for Imaclim-Country

2 Formulary of Imaclim-Country

2.1 General equation framework

The modeling framework boils down to a set of simultaneous equations to solve on each time frame from the base year until the time horizon studied, such as the set of equations 1 which distinguishes three kinds of component: $(x_i)_{i\in[1...k]}$ are the variables computed by the model, which represent the endogenous elements of the projected energy-economy picture, $(\phi_i)_{i\in[1...l]}$ are a set of exogenous parameters (some of them are calibrated at the base year and the other are non-calibrated and come from external sources) and $(f_i)_{i\in[1...k]}$ are a set of exogenous functions. The f_i constraints are of two different natures: one subset of equations describes the accounting constraints that are necessarily verified to ensure the accounting system is properly balanced and the other subset translates the technical and economic choices. The equations of the model could be seen in different blocks as well: price system and income generation, institutional sectors accounts, production and households' consumption trade-offs, and market balances. Thereafter, we will detail the complete set of equations following each block.

$$\forall i \in \ [1 \ldots k], f_i(x_1, \ldots, x_k, \phi_1, \ldots, \phi_l) = 0$$

In this set of equations, the calibrated parameters are identified with an over-line, the initial value for the resolution on each time frame with exponent 0 and the base year variables with exponent BY. Although equations are written in a generalized n-goods and m-households' groups format.

2.2 Growth engine

In its basic version, Imaclim-Country projects the economy in the medium to long run in successive steps of resolution and relies on the method of comparative statics. Hereafter, t gives the time step between the base year and the year of resolution. The growth engine is basically exogenous and technical progress is implemented through factor augmenting coefficients (see Equations 33, 34, and 35). It combines several drivers:

- The total population and active population growth:

$$N = (1 + \delta_N)^t \cdot N_0 \tag{1}$$

$$NL = (1 + \delta_{NL})^t \cdot N_0 \tag{2}$$

- The implicit capital accumulation computed through a proportional link between total fixed capital consumption and the current level of total investment in capital good (see Equation 62)

2.3 Producer and Consumer Prices

p_{Y_i} the producer price of good i is built following the cost structure of the production of good i , that is as the sum of intermediate consumptions, labor costs, capital costs, a tax on production, and a constant mark-up rate (corresponding to the net operating surplus):

$$p_{Y_i} = \sum_{j=1}^{n} p_{IC_i} \cdot \alpha_{ij} + p_{L_i} \cdot l_i + p_{K_i} \cdot k_i + \overline{\tau_{Y_i}} \cdot p_{Y_i} + \overline{\pi_i} \cdot p_{Y_i} \tag{3}$$

The p_{M_i} price of imported good j is good-specific and the international composite good is the *numéraire* of the model; its price is assumed constant and equal to unity.

$$p_{M_{comp}} = p_{M_{comp_0}} = 1 \tag{4}$$

The prices of others goods evolve according to an exogenous rate $\delta_{p_{M_i}}$:

$$p_{M_i} = \left(1 + \delta_{p_{M_i}}\right)^t \cdot p_{M_{i_0}} \tag{5}$$

The $\delta_{p_{M_i}}$ parameters is used to simulate alternative world energy prices scenarios.

The average price p_i of the resource of good i is the weighted average of the two previous prices:

$$\frac{p_{Y_i} \cdot Y_i + p_{M_i} \cdot M_i}{Y_i + M_i} \tag{6}$$

The domestic and foreign varieties of the energy goods are indeed assumed homogeneous: the alternative assumption of product differentiation, adopted by many computable general equilibrium (CGE) model through their use of an Armington specification for international trade (Armington, 1969), has the disadvantage of creating 'hybrid' good varieties, whose volume unit is independent from that of the foreign and national varieties they hybridize; this forbids to maintain an explicit accounting of the physical energy flows and thus an energy balance. For the sake of simplicity, the non-energy goods are treated similarly. p_{IC_i} the price of good i consumed in the production of good j is equal to the resource price of good i plus trade and transport margins, specific margins, a domestic excise on oil products (energy product tax, EnT) , an aggregate of other excise taxes and a carbon tax which equal zero for this paper.

$$p_{IC_i} = p_i \cdot (1 + \tau_{CM_i} + \tau_{TM_i} + \overline{\tau_{SMIC_i}}) + \overline{t_{EnT_{IC_i}}} + \overline{t_{OPT_i}} + t_{c_{IC}} \cdot \gamma_{IC_{ij}} \tag{7}$$

The consumer price of good i for households p_{C_i}, public administrations (p_{G_i}) and investment (p_{I_i}), and the export price of good i (p_{X_i}), are constructed similarly and only differ on whether they are subject to value-added tax (the same rate is applied to all consumptions of one good) and the carbon tax or not. The latter tax applies to household prices only, as national accounting makes households the only final consumer of energy goods.

$$p_{C_i} = [p_i \cdot (1 + \tau_{CM_i} + \tau_{TM_i} + \overline{\tau_{SMC_i}}) + \overline{t_{EnT_{FC_i}}} + \overline{t_{OPT_i}} + t_{c_{FC}} \cdot \gamma_{FC_{ij}}] \cdot (1 + \overline{\tau_{VAT_i}}) \tag{8}$$

$$p_{G_i} = [p_i \cdot (1 + \tau_{CM_i} + \tau_{TM_i} + \overline{\tau_{SMG_i}}) + \overline{t_{OPT_i}}] \cdot (1 + \overline{\tau_{VAT_i}}) \tag{9}$$

$$p_{I_i} = [p_i \cdot (1 + \tau_{CM_i} + \tau_{TM_i} + \overline{\tau_{SMI_i}}) + \overline{t_{OPT_i}}] \cdot (1 + \overline{\tau_{VAT_i}}) \tag{10}$$

$$p_{X_i} = p_i \cdot (1 + \tau_{CM_i} + \tau_{TM_i} + \overline{\tau_{SMX_i}}) + \overline{t_{OPT_i}} \tag{11}$$

Trade margins τ_{CM_i} and transport margins τ_{TM_i} , identical for all intermediate and final consumptions of good i, are calibrated at the reference equilibrium and kept constant-except those on the productions aggregating transport and trade activities (hereafter indexed TRADE and TRANS), which are simply adjusted, in the derived equilibrium, to have the two types of margins sum up to zero :

$$\sum_{j=1}^{n} \tau_{CM_{TRADE}} \cdot p_{TRADE} \cdot \alpha_{TRADE_j} + \tau_{CM_{TRADE}} \cdot p_{TRADE} \cdot (C_{TRADE} + G_{TRADE} + I_{TRADE} + X_{TRADE}) + \sum_{i \neq TRADE}^{n} \sum_{j=1}^{n} \overline{\tau_{CM_i}} \cdot p_i \cdot \alpha_{ij} \cdot Y_j + \sum_{i \neq TRADE} \overline{\tau_{CM_i}} \cdot p_i \cdot (C_i + G_i + I_i + X_i) = 0 \tag{12}$$

and similarly:

$$\sum_{j=1}^{n} \tau_{CM_{TRANS}} \cdot p_{TRANS} \cdot \alpha_{TRANS_j} + \tau_{CM_{TRANS}} \cdot p_{TRANS} \cdot (C_{TRANS} + G_{TRANS} + I_{TRANS} + X_{TRANS}) + \sum_{i \neq TRANS}^{n} \sum_{j=1}^{n} \overline{\tau_{CM_i}} \cdot p_i \cdot \alpha_{ij} \cdot Y_j + \sum_{i \neq TRANS} \overline{\tau_{CM_i}} \cdot p_i \cdot (C_i + G_i + I_i + X_i) = 0 \tag{13}$$

Labor costs are equal to the net wage ω_i plus payroll taxes that are levied based on rates

τ_{LT_i} calibrated at base year:

$$p_{L_i} = (1 + \overline{\tau_{LT_i}}) \cdot \omega_i \tag{14}$$

In some versions, τ_{LT} adjusts following either tax revenues, or any other public budget constraint (see Section 2.9).

The net wage ω_i varies as the average wage Ω :

$$\omega_i = \frac{\Omega}{\Omega_{BY}} \cdot \omega_{i_{BY}} \tag{15}$$

The average wage Ω, subject to variations dictated by the labor market, being defined as:

$$\Omega = \frac{\sum_{i=1}^{n} \omega_i \cdot l_i \cdot Y_i}{\sum_{i=1}^{n} l_i \cdot Y_i} \tag{16}$$

The cost of capital is understood as the cost of the 'machine' capital (see the description of the production trade-offs). It is obtained as the average price of investment goods.

$$p_K = \frac{\sum_{i=1}^{n} p_{I_i} \cdot I_i}{\sum_{i=1}^{n} I_i} \tag{17}$$

The consumer price index *CPI* is computed following Fisher, i.e. as the geometric mean of the Laspeyres index and the Paasche index.

$$CPI = \sqrt{\frac{\sum_{i=1}^{n}(p_{C_i} \cdot C_{i_0}) \cdot \sum_{i=1}^{n}(p_{C_i} \cdot C_i)}{\sum_{i=1}^{n}(p_{C_{i_0}} \cdot C_{i_0}) \cdot \sum_{i=1}^{n}(p_{C_{i_0}} \cdot C_i}})} \tag{18}$$

2.4 Households

Households can be disaggregated into m classes (index $h, h \in [1, m]$) to take into account income structures and eventually behaviors and adaptation capacities that can vary significantly from one household to the next. If there is no information on, or no use for household disaggregation The current Imaclim-ARG version does not inform any disaggregation and has a single consumption group for households. For the sake of a generalized formulary, we maintain the h index.

2.4.1 Income formation, savings and investment decision

The gross primary income of class h, or revenue before tax RBT_h is defined as the addition and the subtraction of the following terms:

- A share ω_{L_h} of the sum of aggregate endogenous net wage income $\omega_i\, l_i\, Y_i$, which varies with the number of active people employed in each class.
- A share ω_{K_h} of the fraction of 'capital income'(the gross operating surplus of national accounting) that goes to households, GOS_H (Equation 21). ω_{K_h} are exogenous and, if applied, their calibration is based on *household consumption surveys* and the economic account table.
- Social transfers, in three aggregates: both the pensions benefits (R_{P_h}) and the unemployment benefits (R_{U_h}) following the average wage variation to BY, and the other social benefits (R_{OSB_h}) following the evolution of GDP per capita (N).
- An exogenous share ω_{OT_h} of residual transfers OT_H , which correspond to the sum of "other current transfers" and "capital transfers".
- A 'debt service' $\overline{i_H}\, D_h$, which corresponds to property income (interests, dividends, real estate revenues, etc.). This service is the product of the households' net debt D_h , the evolution of which is explained below (Equation 26), and an effective interest rate $\overline{i_H}$ calibrated at BY.

Hence,

$$RBT_h = \omega_{L_h} \sum_{i=1}^{n} \omega_i l_i Y_i + \overline{\omega_{K_h}}\, GOS_H + R_{P_{h_{BY}}} \cdot \frac{\omega}{\omega_{BY}} + R_{U_{h_{BY}}} \cdot \frac{\omega}{\omega_{BY}} + R_{OSB_{h_{BY}}} \cdot \frac{\frac{GDP}{N}}{\frac{GDP_{BY}}{N_{BY}}} + \overline{\omega_{OT_h}}\, OT_H - i_h\, D_h \tag{19}$$

with in particular OT_H and GOS_H defined as constant shares ω_{OT_H} and ω_{K_H} of OT (equation (60)) and GOS (Equation 36) :

$$OT_H = \overline{\omega_{OT_H}}\, OT \tag{20}$$

$$GOS_H = \overline{\omega_{K_H}}\, GOS \tag{21}$$

The gross disposable income R_h of class h is obtained by subtracting from RBT_h the income tax T_{I_h} levied at a constant average rate (Equation 43), and other direct taxes T_{D_h} that are indexed on CPI (Equation 44). R_{EXP_h}, the consumed income of class h, is inferred from disposable income by subtracting savings which is a proportion of the gross disposable income. The savings rate τ_{S_h} is calibrated at BY.

$$R_h = RBT_h - T_{I_h} - T_{D_h} \tag{22}$$

$$R_{EXP_h} = (1 - \overline{\tau_{S_h}})\, R_h \tag{23}$$

A further exploration of the data available in the economic account table gives households' investment $GFCF_h$ (Gross Fixed Capital Formation) as distinct from their savings; $GFCF_h$ is assumed to follow the simple rule of a fixed ratio to gross disposable income (Equation 25). The difference between savings and investment gives the self-financing capacity of class h: AFC_h.

$$\frac{GFCF_h}{R_h} = \frac{GFCF_{h_0}}{R_{h_0}} \tag{24}$$

$$AFC_h = \overline{\tau_{S_h}} R_h - GFCF_h \tag{25}$$

The evolution of AFC_h can then be used to estimate the evolution of net debt D_h . The computation is based on the simple assumption of a gradual wedge of AFC at each step of resolution.

$$D_h = D_{h_0} + \frac{\delta_{t+1-t}}{2} \left(AFC_{h_0} - AFC_h\right) \tag{26}$$

2.4.2 Consumption

The consumption of households, for each good i, has been defined, without resorting to any explicit utility function, as the sum of a exogenous basic needs, common to all classes, and a consumption in excess of this need that varies according to some income elasticity σ_{CR_i} , and some price-elasticity σ_{CP_i} . For the time being these elasticities come from literature reviews.

$$for\, i \neq composite, \quad C_{i_h} = \beta_{i_h} C_{i_{h_0}} + \left(1 - \beta_{i_h}\right) \left(\frac{p_{C_i}}{CPI} \cdot \frac{1}{p_{C_{i_0}}}\right)^{\sigma_{CP_i}} \left(\frac{R_{EXP_h}}{CPI} \cdot \frac{1}{R_{EXP_{h_0}}}\right)^{\sigma_{CR_i}} C_{i_{h_0}} \tag{27}$$

where β_{i_h} represents the share of the reference consumption of class h that corresponds to a basic need, and with prices indexed in the same way as the consumptions they value.

For a given good i, Equation 27 can be substituted, at each time step, by exogenous information coming from bottom-up models. For example, we can inform exogenous trajectories of energy consumption for households.

The demand for the composite good (that aggregates the rest of the economy) of class h is then simply defined as the balance of the class's consumed income-which amounts to imposing a binding budget constraint.

$$C_{comp_h} = R_{EXP_h} - \sum_{j \neq comp} \left(p_j \, C_j\right) \tag{28}$$

2.5 Production (institutional sector of firms)

2.5.1 Gross disposable income and investment decision

Similar to households, the firms' disposable income R_F is defined as the addition and subtraction of:

- an exogenous share ω_{K_S} of capital income i.e. GOS (Equation 36),
- a 'debt service' (interests, dividends) $i_F \, D_F$ and served at an interest rate i_F calibrated at BY,
- corporate tax payments T_F ,
- and an exogenous share ω_{OT_F} of other transfers OT, which are assumed a constant share of GDP (Equation 60).

$$R_F = \overline{\omega_{K_S}} \, GOS - i_F \, D_F - T_F + \overline{\omega_{OT_F}} \, OT \tag{29}$$

The ratio of the gross fix capital formation of firms $GFCF_F$ to their disposable income R_F is assumed constant; same as for households and in accordance with national accounting their self-financing capacity AFC_F then arises from the difference between R_F and $GFCF_F$. The net debt of firms D_S is then calculated from their AFC_F on the same reasoning as that applied to households.

$$\frac{GFCF_F}{R_F} = \frac{GFCF_{F_0}}{R_{F_0}} \tag{30}$$

$$AFC_F = R_F - GFCF_F \tag{31}$$

$$D_F = D_{F_0} + \frac{\delta_{t+1-t}}{2} \left(AFC_{F_0} - AFC_F\right) \tag{32}$$

2.5.2 Production trade-offs

The production trade-offs, which are the subject of a specific publication (Ghersi and Hourcade, 2006), assume technical asymptotes that constrain the unit consumptions of factors above some floor values. The restrictive assumption is made that the variable shares of the unit consumptions of each factors (secondary inputs, labor and capital) are substitutable according to a CES specification: the existence of a fix share of each of these consumptions implies that the elasticities of substitution of total unit consumptions (sum of the fix and variable shares) are not fixed, but decrease as the consumptions approach their asymptotes. Under these assumptions and constraints, the minimization of unit costs of production leads to a formulation of the unitary consumptions of secondary factors α_{ji}, of labor l_i and of capital k_i which can be written as the sum of the floor value and a consumption above this value. The latter corresponds to the familiar expression of conditional factor demands of a CES production function with an elasticity of σ_i (the coefficients of which, IC_{ij}, $\lambda_{L_{i_0}}$ and $\lambda_{K_{i_0}}$, are calibrated at BY).

$$\alpha_{ji} = \left[\beta_{IC_{ji}}\, \alpha_{ji_0} + \left(\frac{\lambda_{ji}}{p_{IC_{ji}}}\right)_i^{\sigma} \left(\sum_{j=1}^{n} \lambda_{ji}{}_i^{\sigma}\, p_{IC_{ji}}{}^{1-\sigma_i} + \lambda_{L_i}{}_i^{\sigma} \frac{p_{L_i}}{(1+\Phi_i)^t}^{1-\sigma_i} + \lambda_{K_i}{}_i^{\sigma}\, p_{K_i}{}^{1-\sigma_i} \right)^{-\frac{1}{\rho_i}} \right] \quad (33)$$

$$l_i = \left[\beta_{L_i}\, l_{i_0} + \left(\frac{\lambda_{L_i}}{p_{L_i}}\right)_i^{\sigma} \left(\sum_{j=1}^{n} \lambda_{ji}{}_i^{\sigma}\, p_{IC_{ji}}{}^{1-\sigma_i} + \lambda_{L_i}{}_i^{\sigma} \frac{p_{L_i}}{(1+\Phi_i)^t}^{1-\sigma_i} + \lambda_{K_i}{}_i^{\sigma}\, p_{K_i}{}^{1-\sigma_i} \right)^{-\frac{1}{\rho_i}} \right] \quad (34)$$

$$k_i = \left[\beta_{K_i}\, k_{i_0} + \left(\frac{\lambda_{K_i}}{p_{K_i}}\right)_i^{\sigma} \left(\sum_{j=1}^{n} \lambda_{ji}{}_i^{\sigma}\, p_{IC_{ji}}{}^{1-\sigma_i} + \lambda_{L_i}{}_i^{\sigma} \frac{p_{L_i}}{(1+\Phi_i)^t}^{1-\sigma_i} + \lambda_{K_i}{}_i^{\sigma}\, p_{K_i}{}^{1-\sigma_i} \right)^{-\frac{1}{\rho_i}} \right] \quad (35)$$

where for convenience:

$$\rho_i = \frac{\sigma_i - 1}{\sigma_i}$$

The exogenous labor productivity improvements Φ_i is implemented as factor augmenting productivity gains. It makes it possible to drive changes in production patterns to mimic specific economic scenarios.

Let emphasize again that the 'cost of capital' p_K entering the trade-offs is 'stricto sensu' the price of 'machine capital', i.e. equal to a simple weighted sum of the investment prices of immobilized goods (Equation 17), and unrelated to the interest rates charged on financial markets: on the one hand production trade-offs are based upon the strict cost of inputs, including that of physical capital k_i (calibrated on the consumption of fixed capital of the input-output table); on the other hand, regardless of this arbitrage, the firm's activity and a rule of self-investment ($GFCF_F$, Equation 30) lead to a change in financial position D_F , whose service is not assumed to specifically weigh on physical capital as an input.

A peculiarity of the Imaclim-Country modeling is that these results intensities of factors can be substituted by information coming from technical-economic models. For example, we can inform the capital or labor intensity for a sector i such as the electricity sector, at different time steps of the resolution, just as we can inform the energy intensities of all sectors of the economy. Thanks to the hybridization procedure, there is consistency in embedding an energy intensity calculated on volumes in the Imaclim-Country model. In addition, it is also possible to directly inform volumes. In this case, the intensity will be determined endogenously as defined.

2.5.3 Gross operating surplus

Trade-offs in the i productions, constant rates of operating margin π_i and specific margins τ_{SM} determine the gross operating surplus (GOS) :

$$GOS = \sum_{i=1}^{n} \left(p_{K_i}\, k_i\, Y_i + \overline{\pi_i}\, p_{Y_i}\, Y_i \right) + SM \tag{36}$$

This GOS, that corresponds to capital income, is split between agents following constant shares (calibrated at BY). By construction, the specific margins on the different sales SM sum to zero at BY (this is a constraint of the hybridization procedure by building the database), however they do not sum to zero in runs, their constant rates being applied to varying prices. Their expression is then:

$$SM = \sum_{i} \left(\sum_{j} \overline{\tau_{SMIC_{ij}}}\, p_i\, \alpha_{ij}\, Y_j \; + \; \overline{\tau_{SMC_i}}\, p_i\, C_i + \overline{\tau_{SMG_i}}\, p_i\, G_i \; + \overline{\tau_{SMX_i}}\, p_i\, X_i \right) \tag{37}$$

2.6 Public administrations

2.6.1 Tax, social security contributions and fiscal policy

Tax and social security contributions give the larger share of government resources. In most versions of the model, tax rates and excise taxes other than the carbon tax and social contributions are supposed constant, and the various tax revenues are defined by applying these rates to their respective bases:

$$T_Y = \sum_{i=1}^{n} \overline{\tau_{Y_i}}\, p_{Y_i}\, Y_i \tag{38}$$

$$T_{En} = \sum_{i=1}^{n} \sum_{j=1}^{n} \overline{t_{EnT\,IC_{ji}}}\, \alpha_{ji}\, Y_i + \sum_{i=1}^{n} \overline{t_{EnT\,FC_i}}\, (C_i + G_i + I_i) \tag{39}$$

$$T_{OP} = \sum_{i=1}^{n} \sum_{j=1}^{n} \overline{t_{OPT\,j}}\, \alpha_{ji}\, Y_i + \sum_{i=1}^{n} \overline{t_{OPT\,i}}\, (C_i + G_i + I_i) \tag{40}$$

$$T_{VA} = \sum_{i=1}^{n} \frac{\overline{\tau_{VAT_i}}}{1 + \overline{\tau_{VAT_i}}} \left(p_{C_i}\, C_i + p_{G_i}\, G_i + p_{I_i}\, I_i\right) \tag{41}$$

$$T_F = \overline{\tau_{T_F}}\, GOS_F \tag{42}$$

$$T_{I_h} = \overline{\tau_{TI_h}}\, RBT_h \tag{43}$$

$$T_{D_h} = CPI \cdot T_{D_{h_0}} \tag{44}$$

Fiscal revenues of the carbon tax T_{carb} (if applied) and the sum of labor tax (social insurance contributions and benefits) T_L are computed following the same logic:

$$T_L = \tau_{LT} \sum_{i=1}^{n} w_i\, l_i\, Y_i \tag{45}$$

$$T_{carb} = \sum_{i=1}^{n} \sum_{j=1}^{n} t_{carb_{IC}}\, \gamma_{IC_{ji}}\, \alpha_{ji}\, Y_i + \sum_{i=1}^{n} t_{carb_{FC}}\, \gamma_{FC_i}\, C_i \tag{46}$$

The carbon tax on intermediate consumptions ($t_{carb_{IC}}$) and on final consumptions ($t_{carb_{FC}}$) are exogenous (see Section).

2.6.2 Gross disposable income, public spending, investment and transfers

Similar to households and firms (following the logic prevailing in the economic account table), the gross disposable income of public administrations R_G is the sum of taxes and social contributions, of exogenous shares ω_{K_G} of GOS and ω_{OT_G} of 'other transfers' OT , from which are subtracted public expenditures $p_G\, G$, a set of social transfers R_P , R_U and R_{OSB} , and a debt service $i_G\, D_G$:

$$R_G = T + \overline{\omega_{K_G}}\, GOS + \overline{\omega_{OT_G}}\, OT - \sum_{i=1}^{n} p_{G_i}\, G_i - R_P - R_U - R_O - i_G\, D_G \tag{47}$$

Public expenditures $p_G\,G$ are assumed to keep pace with national income, and therefore are constrained as a constant share of GDP:

$$\frac{\sum_{i=1}^{n} p_{G_i}\,G_i}{GDP} = \frac{\sum_{i=1}^{n} p_{Gi_0}\,G_{i_0}}{GDP_0} \tag{48}$$

Social transfers R_P , R_U and R_O are the sum across household classes of the transfers defined as components of their before-tax disposable income (Equation 19) :

$$R_P = \sum_{h=1}^{m} \overline{N_{P_h}}\,\rho_{P_h} \tag{49}$$

$$R_U = \sum_{h=1}^{m} N_{U_h}\,\rho_{U_h} \tag{50}$$

$$R_{OSB} = \sum_{h=1}^{m} \overline{N_h}\,\rho_{O_h} \tag{51}$$

With *per capita* transfers ρ_{P_h} , ρ_{U_h} and ρ_{O_h} indexed on the average net wage:

$$\forall\ K \in [P, U, OSB],\ \forall\ h \in [1, m] \quad \rho_{K_h} = \frac{w}{w_0}\,\rho_{K_{h_0}} \tag{52}$$

At last, the interest rate i_G of public debt evolves as do i_H and i_F (Equation **??**). Public investment $GFCF_G$, same as public expenditures $p_G\,G$, is supposed to mobilize a constant share of GDP. Subtracting it from R_G gives AFC_G , which determines the variation of the public debt:

$$\frac{GFCF_G}{GDP} = \frac{GFCF_{G_0}}{GDP_0} \tag{53}$$

$$AFC_G = R_{GD_G} - GFCF_G \tag{54}$$

$$D_G = D_{G_0}\frac{\delta_{t+1-t}}{2}\,(AFC_{G_0} - AFC_G) \tag{55}$$

2.7 "Rest of the World"

2.7.1 Balance of Trade

Concerning international trade the assumption is made of an open economy whose weight does not affect world prices: global import prices p_M retain their relative values (Equation 5). Then the ratio of imports to domestic production on the one hand, and the 'absolute' exported quantities on the other hand, are elastic to the terms of trade, according to fixed good-specific elasticities :

$$\frac{M_i}{Y_i} = (1+\delta_{M_i})^t \cdot \frac{M_{i_0}}{Y_{i_0}} \left(\frac{p_{M_{i_0}}}{p_{Y_{i_0}}} \frac{p_{Y_i}}{p_{Mi}} \right)^{\sigma_{M p_i}} \tag{56}$$

$$\frac{X_i}{X_{i_0}} = (1+\delta_{X_i})^t \cdot \left(\frac{p_{X_{i_0}}}{p_{M_{i_0}}} \frac{p_{M_i}}{p_{X_i}} \right)^{\sigma_{X p_i}} \tag{57}$$

The different treatment of imports and exports merely reflects the assumption that, notwithstanding the evolution of the terms of trade, import volumes rise in proportion to economic activity (domestic production), while exports do not (global demand is assumed constant). It implies, however, that improved terms of trade do not necessarily mean an improvement in the trade balance, depending on the concomitant variations of activity.

Eventually, exports are impacted by global economic growth, independently of terms of trade variations. This is captured by assuming an exogenous rate of growth of exports δ_{X_i}. Usually, for non-energy goods, this parameter follows world growth assumption over time. Outsourcing and relocations can be captured by the exogenous parameter δ_{M_i}.

For hybrid goods in volume, Equations 56 and 57 can be substituted, at each time step, by exogenous information coming from bottom-up models. For example, we can inform the full exogenous trajectories of energy trade.

2.7.2 Capital flows and self-financing capacity

Capital flows from and to the ROW are not assigned a specific behavior, but are simply determined as the balance of capital flows of the three national institutional sectors (households, firms, public administrations) to ensure the balance of trade accounting. This assumption determines the self-financing capacity of the ROW, which in turn determines the evolution of D_{ROW} , its net financial debt:

$$AFC_{ROW} = \sum_{i=1}^{n} p_{M_i} M_i - \sum_{i=1}^{n} p_{X_i} X_i + \sum_{K=H,S,G}^{n} i_K D_K - \sum_{K=H,S,G}^{n} OT_K \tag{58}$$

$$D_{ROW} = D_{ROW_0} + \frac{\delta_{t+1-t}}{2} \left(AFC_{ROW_0} - AFC_{ROW} \right) \tag{59}$$

By construction the self-financing capacities (*AFC*) of the four agents clear (sum to zero), and accordingly the net positions, which are systematically built on the AFCs, change from a position in which they are strictly compensating each other to another such position. Indeed a nil condition on the sum of net positions could be substituted to Equation 59 without impacting the results of the model. The hypothesis of a systematic 'compensation' by the ROW of the property incomes of national agents without any reference to its debt D_{ROW} may seem crude, but in fine only replicates the method of construction of the economic account table. Indeed, in the no-policy equilibrium the effective interest rate of the ROW (ratio of net debt to its property income), which ultimately results from a myriad of debit and credit positions and from the corresponding capital flows, is negative, so unworkable for modeling purposes.

To conclude, as previously mentioned other transfers OT ("other current transfers " and "capital transfers ") are defined as a fixed share of GDP :

$$\frac{OT}{GDP} = \frac{OT_0}{GDP_0} \tag{60}$$

2.8 Market balances

2.8.1 Goods markets

Goods market clearing is a simple accounting balance between resources (production and imports) and uses (households and public administrations' consumption, investment, exports). Thanks to the process of hybridization, this equation is written in volume for hybrid goods. For energy goods it is written in *Mtoe*for energy goods and consistent with the energy balance used (the G and I of these goods are nil by definition).

$$Y_i + M_i = C_i + G_i + I_i + X_i \tag{61}$$

2.8.2 Investment and capital flows

Contrary to standard CGE models, Imaclim-Country does not represent explicit capital markets, which is admissible given the nature of static exercises conducted. The capital-investment balance is "demand-driven". As previously highlighted, productive sectors arbitrate capital consumption according to prices of equipment, and not according to the return on capital.

Then, the capital immobilized in all productions is supposed homogeneous, and all its components vary as the total consumption of fixed capital:

$$\frac{I_i}{\sum_{j=1}^{n} k_j \cdot Y_j} = \overline{\beta_i} = \frac{I_{i_0}}{\sum_{j=1}^{n} k_{j_0} \cdot Y_{j_0}} \tag{62}$$

In the meantime, the assumption is made of a single investment good in the economy as a weighted sum of different goods calibrated at base year.

Commonly, in Imaclim-Country and other CGE models, the investment is a vector: it informs for each sector its investment within the overall economy. It makes it difficult to undertake a robust evaluation of the impacts on structural changes in contrasted investment plans for a specific sector j. This is why a specific modeling approach can be set up by describing a complete *investment matrix* I_{ij} at calibration (thanks to capital breakdown literature). Instead of being of vector dimension, Equation 62 is then of matrix dimension, but the modeling constraint remains the same. However, for each time step, it allows us to specify the demand for investment -by sector- generated by the sector j. This specification is used to model contrasted investment in the power sector's technologies.

Furthermore, the investment supply adapts to the demand. Capital formation from firms completes households and public contribution to satisfy that demand, and, balance investment flows.

$$GFCF_H + GFCF_F + GFCF_G = \sum_{i=1}^{n} p_{I_i} \cdot I_i \tag{63}$$

Eventually, the investment balance, together with households saving rate and public expenses, imposes the external or trade balance which is endogenous in the model.

2.8.3 Employment

The labor market conditions, results from the interplay between labor demand from the production systems, equal to the sum of their factor demands $l_i\,Y_i$, and of labor supply from households. As part of key structural assumptions, the model allows for a strictly positive unemployment rate u and the market balance writes:

$$(1 - u)\,\overline{N}L = \sum_{i=1}^{n} l_i\,Y_i \tag{64}$$

Rather than explicitly describe labor supply behavior, the model infers changes in u following a wage curve, which describes an empirical correlation between the average real wage Ω and the unemployment rate u, characterized by an constant elasticity σ_{w_u}. The underlying intuition is that any increase in unemployment creates a downward pressure on wages, which is indeed interpretable in terms of either bargaining power, or efficiency wage:

$$\frac{\Omega}{\beta_{w_{CPI_i}} \cdot CPI + (1 - \beta_{w_{CPI_i}})} = \Omega_0 \cdot \left(\frac{u}{u_0}\right)^{\sigma_{w_u}} \tag{65}$$

There is also the possibility of setting a sectoral wage curve. It takes the same formulation as the previous equation but uses the net wage of sector ω_i. In this configuration, Equation 15 is removed from the system. The indexation parameter $\beta_{w_{CPI_i}}$ gives a flexibility to describe a wage curve based either on nominal wage ($\beta_{w_{CPI_i}} = 0$) or real wage ($\beta_{w_{CPI_i}} = 1$). For all values included between 0 and 1, we assume that only a share of wage can be bargain based on real wage. The underlying intuition is that any increase in unemployment creates a downward pressure on wages, which is indeed interpretable in terms of either bargaining power, or efficiency wage. According to the openness of the economy (or the sectors for a sectoral wage curve) at the international scale and its mobility, the bargain lies between real wage and nominal wage.

If different household classes are represented, changes in employment corresponding to the evolution of u are then split between the classes according to their specific unemployment u_h :

$$u_h = u_{h_0} \frac{u}{u_0} \tag{66}$$

2.9 Carbon tax policies

The model is specifically designed to study carbon tax policies in the medium to long run by generating policy-constrained projections. In the model, implementing a carbon tax amounts to adding a shock on energy prices proportional to their carbon content at the time horizon studied.

The implementation of carbon prices[1] ($t_{carb_{IC}}$ and t_{carb_C}) increase intermediate consumption prices (Equation 7) and purchaser's price for households (Equation 7) in proportion of the emission factor of the energy good i consummed by sector j ($\gamma_{IC_{ij}}$), and households (γ_{C_i}) [2].

In the present model version, Imaclim-Country can simulate different uses of the carbon tax revenues T_{carb}, that can be split in three categories. The carbon tax revenues is either used:

- to only feed public budget, or
- to directly compensate domestic agents: firms and/or households, or
- to reduce existing other taxes rates.

The recycling options can be combined: a share of the carbon tax revenues may be directly returned to domestic agents, and the remained part is then used to reduce other taxes rates.

Direct compensations can be computed by the following constraints:

[1]The framework can accommodate sector specific carbon prices.
[2]The γ emission factor is calibrated at BY but can be adapted exogenously if required.

- Lump-sum transfer to households
  A share (δ_{LS_H}) of the carbon tax revenues are directly transferred to households while maintaining neutral policy budget:

$$LS_H = \delta_{LS_H} \cdot T_{carb} \text{ with } \delta_{LS_F} \in [0,1] \tag{67}$$

- Lump-sum transfer to firms
  A share (δ_{LS_F}) of the carbon tax revenues are directly transferred to firms while maintaining neutral policy budget:

$$LS_F = \delta_{LS_F} \cdot T_{carb} \text{ with } \delta_{LS_F} \in [0,1] \tag{68}$$

The shares δ_{LS_H} and δ_{LS_F} can be fixed exogenously as a parameters or can correspond to a proportion of variables, endogenously. For no direct transfers to agents, the shares are set up to zero.

Carbon tax revenues can be used to alleviate different existing taxation. We focus on a standard case: the reduction of the labor tax that can be set up under different constraints:

- All carbon tax revenues (net of direct compensation if also applied) are used to reduce the labor taxation while maintaining neutral policy budget. Thus, under this recycling option, the sector specific rates of labor taxes $\overline{\tau_{T_L}}$ is alleviated by the same coefficient Δ_{T_L}:

$$T_{carb} - LS_H - LS_F = \Delta_{T_L} \sum_i^n \omega_i \cdot l_i \cdot Y_i \tag{69}$$

$$\text{with } \overline{\tau_{T_L}} = \tau_{T_L} + \Delta_{T_L} \tag{70}$$

- The level of labor tax reduction is determined while imposing a specific budget constraint. Thus, the cut Δ_{T_L} is endogenously calculated to satisfy the budget condition.
  Unless otherwise specified, carbon tax implementations are recycled into labor tax reduction while maintaining the government financial capacity constant to GDP variation:

$$\frac{AFG_G}{GDP} = \frac{AFG_{G_0}}{GDP_0} \tag{71}$$

3 The accounting framework

All interactions are synthesised in a large accounting framework composed by two tables.

- The Input-Output table balances the uses and resources of products (see Table 4). Each sector produces one single good so that commodities and activities match and the Input-Ouput table (IOT).

- The national economic accounts table details the primary and secondary distribution of income between representative economic agents (see 5). The model distinguishes four economic agents : *households* (H)[3], *corporate firms* (F), *public administrations* or *government* (G) and the *rest of the world* (ROW).

Imaclim Brazil keeps detail on primary income distribution between each economic agents. Therefore households, firms and government have separated accounts in our model and may have different structural behaviours. This is an extention to traditional framework of CGE models that usually shortcut this aspect by assuming that households eventually own the total endowment of production factors. Households are the only agent to own labour factor.

Furthermore, through secondary income distribution, economic agents break down their income between goods consumption, investment, tax payments and transfers. The model considers a detailed system of taxes and transfers essentially between the triangle of domestic agents (households, firms and public administrations). This will be detailed in the following.

Owing to the split of accounts of institutional agents, Imaclim Brazil also considers the breakdown of total Gross Fixed Capital Formation (GFCF) between agents. It further identifies for each agent the share of income that is not directly invested in GFCF, which is called self-financing capacity (*ACF*). The rest of the world classically interacts with domestic agents through trade of goods and capital balance.

The rest of this section details the equations of the model through different blocks: (i) price system and income generation, (ii) institutional sectors accounts, (iii) production and consumption trade-off, (iv) market balances, (v) growth engine, (vi) carbon tax policies.

The equations of the model are of two quite different natures: one subset of equations describes accounting constraints that are necessarily verified to ensure that the accounting system is properly balanced; the other subset translates various behavioural constraints, written either in a simple linear manner (e.g. households consume a fixed proportion of their income) or in a more complex non-linear way (e.g. the trade-offs of production and consumption). It is these behavioural constraints that ultimately reflect a certain *economic worldview*.

[3]Households can be disaggregated into ten classes of revenues

		Intermediate consumption ($IC_{val_{ij}}$)		Final Consumption (FC_{val_i})				Uses (Use_i)
		Hybrid sectors	***Non-hybrid sectors***	***Households*** (C_{val_i})	***Public administration*** (G_{val_i})	***Investment*** (I_{val_i})	***Exports*** (X_{val_i})	
Intermediate consumption ($IC_{val_{ij}}$)	***Hybrid sectors***	$p_{IC_{ij}} \cdot \alpha_{ij} \cdot Y_j$	$p_{IC_{ij}} \cdot \alpha_{ij} \cdot Y_j$	$p_{C_i} \cdot C_i$	$p_{G_i} \cdot G_i$	$p_{I_i} \cdot I_i$	$p_{X_i} \cdot X_i$	$\sum_j IC_{val_{ij}} + FC_{val_i}$
	Non-hybrid sectors	$p_{IC_{ij}} \cdot \alpha_{ij} \cdot Y_j$	$p_{IC_{ij}} \cdot \alpha_{ij} \cdot Y_j$	$p_{C_i} \cdot C_i$	$p_{G_i} \cdot G_i$	$p_{I_i} \cdot I_i$	$p_{X_i} \cdot X_i$	$\sum_j IC_{val_{ij}} + FC_{val_i}$
Value-added (VA_j)	***Labour income***	$\omega_j \cdot l_j \cdot Y_j$	$\omega_j \cdot l_j \cdot Y_j$					
	Labour Tax	$\tau_{T_L} \cdot \omega_j \cdot l_j \cdot Y_j$	$\tau_{T_L} \cdot \omega_j \cdot l_j \cdot Y_j$					
	Capital income	$p_{K_j} \cdot k_j \cdot Y_j$	$p_{K_j} \cdot k_j \cdot Y_j$					
	Production Tax	$\tau_{T_{Y_i}} \cdot p_{Y_j} \cdot Y_j$	$\tau_{T_{Y_i}} \cdot p_{Y_j} \cdot Y_j$					
	Profit margin	$\pi_j \cdot p_{Y_j} \cdot Y_j$	$\pi_j \cdot p_{Y_j} \cdot Y_j$					
Production (Y_{val_j})		$p_{Y_j} \cdot Y_j$	$p_{Y_j} \cdot Y_j$					
Imports (M_{val_j})		$p_{M_j} \cdot M_j$	$p_{M_j} \cdot M_j$					
Margins ($Marg_j$)	***Trade margins***	$\tau_{CM_j} \cdot p_j \cdot (Y_j + M_j)$	$\tau_{CM_j} \cdot p_j \cdot (Y_j + M_j)$					
	Transport margins	$\tau_{TM_j} \cdot p_j \cdot (Y_j + M_j)$	$\tau_{TM_j} \cdot p_j \cdot (Y_j + M_j)$					
	Specific margins on IC ($SM_{IC_{ij}}$)	$\tau_{SM_{IC_{ij}}} \cdot p_j \cdot \alpha_{ji} \cdot Y_j$	0					
	SpeMarg on C (SM_{C_j})	$\tau_{SM_{C_j}} \cdot p_j \cdot C_j$	0					
	SpeMarg on G (SM_{G_j})	$\tau_{SM_{G_j}} \cdot p_j \cdot G_j$	0					
	SpeMarg (SM_{I_j})	$\tau_{SM_{I_j}} \cdot p_j \cdot I_j$	0					
	SpeMarg (SM_{X_j})	$\tau_{SM_{X_j}} \cdot p_j \cdot X_j$	0					
Taxes (T_j)	***Value-added tax***	$(\frac{\tau_{VAT_j}}{1-\tau_{VAT_j}}) \cdot FC_{val_j}$	$(\frac{\tau_{VAT_j}}{1-\tau_{VAT_j}}) \cdot FC_{val_j}$					
	Energy Tax IC	$t_{EnT_{IC_j}} \cdot \sum_i(\alpha_{ji} \cdot Y_i)$	$t_{EnT_{IC_j}} \cdot \sum_i(\alpha_{ji} \cdot Y_i)$					
	Energy Tax FC	$t_{EnT_{FC_j}} \cdot (C_j + G_j + I_j)$	$t_{EnT_{FC_j}} \cdot (C_j + G_j + I_j)$					
	Other indirect tax on products	$t_{OPT_j} \cdot (\sum_i \alpha_{ji} \cdot Y_i + C_j + G_j + I_j)$	$t_{OPT_j} \cdot (\sum_i \alpha_{ji} \cdot Y_i + C_j + G_j + I_j)$					
Supply (Sup_j)		$Y_{val_j} + M_{val_j} + Marg_j + T_j + T_j$	$Y_{val_j} + M_{val_j} + Marg_j + T_j + T_j$					

Table 4 – IMACLIM-COUNTRY Input-Output table (IOT)

	Firms (F)	Public administration (G)	Households (H)	Rest of world (ROW)
Trade balance	-	-	-	$\sum_i (p_{M_i} \cdot M_i - p_{X_i} \cdot X_i)$
Gross operating surplus (GOS)[4]	$\omega_{K_F} \cdot GOS$	$\omega_{K_G} \cdot GOS$	$\omega_{K_H} \cdot GOS$	-
Labour income	-	-	$\sum_j (\omega_j \cdot l_j \cdot Y_j)$	-
Labour tax (T_L)	-	$\tau_{T_L} \sum_j (\omega_j \cdot l_j \cdot Y_j)$	-	-
Production tax (T_Y)	-	$\sum_j \tau_{T_{Y_j}} \cdot p_{Y_j} \cdot Y_j$	-	-
Energy tax (T_{En})	-	$\sum_j [t_{EnT_{IC_j}} \cdot \sum_i (\alpha_{ji} \cdot Y_i) + t_{EnT_{FC_j}} \cdot (C_j + G_j + I_j)]$	-	-
Other indirect tax (T_{OP})	-	$\sum_j [t_{OPT_j} \cdot (\sum_i \alpha_{ji} \cdot Y_i + C_j + G_j + I_j)]$	-	-
Value-added tax (T_{VA})	-	$\sum_j [(\frac{\tau_{VAT_j}}{1-\tau_{VAT_j}}) \cdot FC_{val_j}]$	-	-
Property income	$-i_F \cdot D_F$	$-i_G \cdot D_G$	$-\sum_h (i_h \cdot D_h)$	$i_F \cdot D_F + i_G \cdot D_G + \sum_h (i_h \cdot D_h)$
Unemployment transfers (U)	-	$-\sum_h (\rho_{U_h} \cdot N_{U_h})$	$\sum_h (\rho_{U_h} \cdot N_{U_h})$	-
Pensions (P)	-	$-\sum_h (\rho_{P_h} \cdot N_{P_h})$	$\sum_h (\rho_{P_h} \cdot N_{P_h})$	-
Other social transfers (O)	-	$-\sum_h (\rho_{O_h} \cdot N_{O_h})$	$\sum_h (\rho_{O_h} \cdot N_{O_h})$	-
Other transfers (OT)	$\omega_{OT_F} \cdot OT$	$\omega_{OT_G} \cdot OT$	$\omega_{OT_H} \cdot OT$	$-(\omega_{OT_F} \cdot OT + \omega_{OT_G} \cdot OT + \omega_{OT_H} \cdot OT)$
Income tax (T_{I_H})	-	$\sum_h (\tau_{T_{I_h}} \cdot RBT_h)$	$-\sum_h (\tau_{T_{I_h}} \cdot RBT_h)$	-
Firm tax (T_F)	$-\tau_{T_F} \cdot GOS_F$	$\tau_{T_F} \cdot GOS_F$	-	-
Other direct tax (T_D)	-	$\sum_h (\tau_{T_{D_h}} \cdot CPI)$	$-\sum_h (\tau_{T_{D_{h_0}}} \cdot CPI)$	-
Gross disposable income (R)	$R_F = \sum$ row below	$R_G = \sum$ row below	$R_H = \sum$ row below	$R_{ROW} = \sum$ row below
Final consumption (FC_{val})	-	$\sum_i G_{val_i}$	$R_{CONS_H} = \sum_h (1 - \tau_{S_h}) \cdot R_h$	-
Gross fixed capital consumption ($GFCF$)	$GFCF_F = \frac{GFCF_{F_0}}{R_{F_0}} \cdot R_F$	$GFCF_G = \frac{GFCF_{G_0}}{GDP_0} \cdot GDP$	$GFCF_H = \sum_h (\frac{GFCF_{h_0}}{R_{h_0}} \cdot R_h)$	-
Expenses for final uses	$GFCF_F$	$\sum_i G_{val_i} + GFCF_G$	$R_{CONS_H} + GFCF_H$	-
Self-financing capacity (AFC)	$AFC_F = R_F - GFCF_F$	$AFC_G = R_G - [GFCF_G + \sum_i G_{val_i}]$	$AFC_H = \sum_h (\tau_{S_h} \cdot R_h - GFCF_h)$	$ACF_{ROW} = -(AFC_F + ACF_G + ACF_H)$
Net financial debt (D)	$D_F = D_{F_0} + \frac{t_{ref}}{2} \cdot (AFC_{F_0} - ACF_F)$	$D_G = D_{G_0} + \frac{t_{ref}}{2} \cdot (AFC_{G_0} - ACF_G)$	$D_H = D_{H_0} + \frac{t_{ref}}{2} \cdot (AFC_{H_0} - ACF_H)$	$D_{ROW} = -(D_F + D_G + D_H)$

Table 5 – IMACLIM-COUNTRY national economic accounts table

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