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RATIONALITY AND AGGREGATION OF PREFERENCES IN AN ORDINALLY FUZZY FRAMEWORK

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ABSTRACT

The paper explores the problem of aggregating ordinally fuzzy individual preferences into ordinally fuzzy social preferences. Using Goguen's ordinal formulation of fuzziness, it is shown that, given certain plausible conditions, the requirement that the society's preferences should satisfy any of several alternative transitivity conditions creates a dilemma: either power in the society is rather unevenly distributed, or the society tends to be indecisive. The problem increases as weaker transitivity conditions are replaced by stronger ones.

Keywords: fuzzy preferences, fuzzy aggregation rule, transitivity.

1. Introduction

The purpose of this paper is to investigate the problem of aggregating fuzzy individual preferences into fuzzy social preferences. While this problem has been considered earlier by several writers (see, among others, Barrett, Pattanaik and Salles (1986), Barrett and Pattanaik (1989), Blin and Whinston (1979), Blin (1974), Dimitrov (1983), Leclerc (1984), Nurmi (1981), Subramanian (1988) and Tanino (1984)), the present paper differs from most earlier contributions in two important respects. First, in contrast to most earlier writers, who use the standard fuzzy set theoretic framework (with 'degrees of belonging' represented by numbers in the interval [0, 1]), we use an ordinal version of fuzziness due to Goguen (1967). Thus, instead of representing degrees of belonging, in a cardinal fashion, by numbers lying between 0 and 1, we just assume an ordering of these degrees. Secondly, instead of concentrating on any single transitivity property for individual and social preferences, we explore the implications of imposing alternative transitivity properties, some being very weak. Since the literature on fuzzy preferences contains a number of transitivity conditions, and, since none of these conditions seems to have any overwhelming intuitive claim to superiority over the others, it seems useful to explore the implications of different transitivity properties in the context of aggregation of preferences. This is what we have tried to do in the present paper. Using Goguen's ordinal formulation of fuzziness, we show that, given certain plausible conditions, the requirement that the society's preferences should satisfy even one of the weaker transitivity conditions considered by us creates a dilemma: either power in the society is rather unevenly distributed, or the society tends to be indecisive. The problem increases as weaker transitivity conditions are replaced by stronger ones.

2. Notation and Definitions

Let X be a set of social alternatives, and let $N = \{1, \ldots, n\}$ index a finite set of individuals constituting the society. Let L ($\#L \geq 2$) be a finite set, ordered by an ordering $\succsim$; the elements of L are to be interpreted as "degrees of belonging". Let $\succ$ denote the asymmetric factor of $\succsim$, and $\sim$ the symmetric factor, and assume that L has a unique $\succsim$-least element d_* and a unique $\succsim$-greatest element d^*. (See Goguen (1967) for the seminal work on an ordinally fuzzy framework, in which incidentally he refers to 'L-sets' rather than 'fuzzy sets'.) A fuzzy binary relation over X, in this context, is defined to be a function $f: X^2 \to L$.

Notation 2.1 Let F be the set of all functions $f: X^2 \to L$ such that:

(2.1) for all $x \in X$, $f(x, x) = d_*$;

(2.2) for all distinct $x, y \in X$, $f(x, y) = d^*$ implies $f(y, x) = d_*$;

(2.3) for all $x, y, z \in X$, $f(x, y) = d^*$ implies $f(x, z) \succsim f(y, z)$, and $f(y, z) = d^*$ implies $f(x, z) \succsim f(x, y)$.

We interpret F as the set of all fuzzy (binary) strict preference relations (FSPR's) over X.

Remark 2.2 (2.1) and (2.2) are familiar conditions, known respectively as irreflexivity and asymmetry. (2.3) is a transitivity condition which is just as compelling. When in the next section we consider a variety of transitivity conditions, some of which are very weak, we find it difficult to be confident, in a general framework, that any particular transitivity condition is the most appropriate to adopt. When however exact strict preference is involved, as in (2.3), matters are different. According to (2.3), if x is exactly better than y, then x must 'fare as well' against z as y against z (in terms of preference in favour); and, if y is exactly better than z, then x must 'fare as well' against z as against y (in terms of preference in favour).

Definition 2.3 A fuzzy aggregation rule (FAR) is a function $g: G^n \to F$, where $\emptyset \neq G \subseteq F$.

Thus, an FAR maps an n-tuple of FSPR's, interpreted as the FSPR's of the individuals of the society, to an FSPR, interpreted as the society's FSPR. We write $P = g(P_1, \ldots, P_n)$, $P' = g(P_1', \ldots, P_n')$, etc.

Definition 2.4 An FAR satisfies:

(2.4.1) unanimity (U) iff, for all $x, y \varepsilon X$ and for all $(P_1, \ldots, P_n) \varepsilon G^n$, there exists $i \varepsilon N$ such that $P_i(x, y) \not\succ P(x, y)$, and there exists $j \varepsilon N$ such that $P(x, y) \not\succ P_j(x, y)$.

(2.4.2) independence of irrelevant alternatives (IIA) iff, for all $x, y \varepsilon X$ and for all $(P_1, \ldots, P_n)$, $(P_1', \ldots, P_n') \varepsilon G^n$, [for all $i \varepsilon N$, $P_i(x, y) \sim P_i'(x, y)$ and $P_i(y, x) \sim P_i'(y, x)$] implies [$P(x, y) \sim P'(x, y)$ and $P(y, x) \sim P'(y, x)$].

Remark 2.5 IIA is the counterpart of the corresponding condition in the exact framework, and U is similar in spirit to the familiar Pareto criterion (see Arrow (1963)).

Notation 2.6 A non-empty subset of N will be called a coalition. Given a coalition $C = \{i_1, \ldots, i_m\}$ (where $i_1 < \ldots < i_m$), $x, y \in X$ and $(P_1, \ldots, P_n) \in G^n$, $P_C(x, y)$ will denote $(P_{i_1}(x, y), \ldots, P_{i_m}(x, y))$. Given $d \in L$, $P_C(x, y) \succsim d$ will denote $P_k(x, y) \succsim d$ for all $k \in C$. Similarly for $P_C(x, y) \succ d$, $P_C(x, y) \sim d$, etc.

3. Transitivity Conditions

The choice of transitivity conditions to be assumed for individual and also social preferences is an important issue in any discussion of aggregation of individual preferences into social preferences. The problem is particularly difficult when one operates in the fuzzy framework, since, in such a framework, one has many transitivity conditions to consider, none of which has any obvious intuitive superiority over the rest. Therefore, instead of confining ourselves to any single transitivity condition, we investigate the implications of a variety of such conditions.

Definition 3.1 An FSPR f over X satisfies:

(3.1.1) restricted max-min transitivity iff for all $x, y, z \in X$, $[f(x, y) \succsim f(y, x)$ and $f(y, z) \succsim f(z, y)]$ implies $[f(x, z) \succsim f(x, y)$ or $f(x, z) \succsim f(y, z)]$;

(3.1.2) quasi-transitivity iff for all $x, y, z \in X$, $[f(x, y) \succ$

$f(y, x)$ and $f(y, z) \succ f(z, y)$] implies $f(x, z) \succ f(z, x)$;

(3.1.3) <u>acyclicity</u> iff there does not exist a sequence x_1, x_2, ..., $x_r \varepsilon X$ $(r > 1)$ such that [$f(x_1, x_2) \succ f(x_2, x_1)$ and ... and $f(x_{r-1}, x_r) \succ f(x_r, x_{r-1})$ and $f(x_r, x_1) \succ f(x_1, x_r)$];

(3.1.4) <u>simple transitivity</u> iff for all $x, y, z \varepsilon X$, [$f(x, y) \succ d_*$ and $f(y, x) = d_*$ and $f(y, z) \succ d_*$ and $f(z, y) = d_*$] implies [$f(x, z) \succ d_*$ and $f(z, x) = d_*$].

<u>Remark 3.2</u> An example, found in Barrett and Pattanaik (1986), involves the following three alternatives: a sum of money m, $m + \delta$ $(\delta > 0)$ and x (unspecified). For δ small, one might easily observe (and without there seeming to be anything irrational about such preferences): $f(m + \delta, m) = d^*$, $f(m, x) = d$ and $f(x, m + \delta) = d'$, where $d^* \succ d \succ d_*$ and $d^* \succ d' \succ d_*$. Nevertheless, these preferences conflict with the familiar condition of max-min transitivity:

(3.1) for all $x, y, z \varepsilon X$, [$f(x, z) \succcurlyeq f(x, y)$ or $f(x, z) \succcurlyeq f(y, z)$];

and also with the weaker transitivity condition used in Barrett, Pattanaik and Salles (1986):

(3.2) for all $x, y, z \varepsilon X$, [$f(x, y) \succ d_*$ and $f(y, z) \succ d_*$] implies $f(x, z) \succ d_*$.

At the same time the above preferences are consistent with each of the transitivity conditions introduced in Definition 3.1. (See Dasgupta and Deb (1988) for a discussion of some of the available transitivity conditions.)

<u>Notation 3.3</u> Let G_1, G_2, G_3 and G_4 be the set of all $f \varepsilon F$ satisfying, respectively, restricted max-min transitivity, quasi-transitivity,

acyclicity and simple transitivity. Let G' be the set of all $f \varepsilon F$ satisfying

(3.3) for all x, y, $z \varepsilon X$, $[f(x, y) = d_*$ and $f(y, z) = d_*]$ implies $f(x, z) = d_*$.

<u>Proposition 3.4</u> $[G_1 \subset G_2 \subset G_3]$ and $[G_2 \cap G' \subset G_4 \cap G']$.

<u>Proof</u> We prove just $G_1 \subseteq G_2$, since the rest is straightforward. Let $f \varepsilon G_1$, let x, y, $z \varepsilon X$, and let $f(x, y) \succ f(y, x)$ and $f(y, z) \succ f(z, y)$. Suppose $f(z, x) \succeq f(x, z)$. Then, by restricted max-min transitivity, $[f(x, z) \succeq f(x, y)$ or $f(x, z) \succeq f(y, z)]$, $[f(y, x) \succeq f(y, z)$ or $f(y, x) \succeq f(z, x)]$ and $[f(z, y) \succeq f(z, x)$ or $f(z, y) \succeq f(x, y)]$. Therefore, one of the following must hold:

$$[f(x, z) \succeq f(x, y) \succ f(y, x) \succeq f(y, z) \succ f(z, y) \succeq f(z, x) \succeq f(x, z)];$$

$$[f(x, z) \succeq f(x, y) \succ f(y, x) \succeq f(y, z) \succ f(z, y) \succeq f(x, y)];$$

$$[f(x, z) \succeq f(x, y) \succ f(y, x) \succeq f(z, x) \succeq f(x, z)];$$

$$[f(x, z) \succeq f(y, z) \succ f(z, y) \succeq f(z, x) \succeq f(x, z)];$$

$$[f(x, z) \succeq f(y, z) \succ f(z, y) \succeq f(x, y) \succ f(y, x) \succeq f(y, z)];$$

$$[f(x, z) \succeq f(y, z) \succ f(z, y) \succeq f(x, y) \succ f(y, x) \succeq f(z, x) \succeq f(x, z)].$$

Since each of these involves a contradiction, $f(x, z) \succ f(z, x)$, and the proof of $G_1 \subseteq G_2$ is complete.

4. The Results

First we give a preliminary result on the 'neutrality' and 'monotonicity' of FAR's.

Proposition 4.1 Let $g: G^n \to F$, where $G_1 \subseteq G$, be an FAR which satisfies U and IIA, let distinct $x, y, z, w \in X$ and let $(P_1, \ldots, P_n)$, $(P_1', \ldots, P_n') \in G^n$. Then, $[P_N(x, y) \succcurlyeq P_N'(z, w)$ and $P_N'(w, z) \succcurlyeq P_N(y, x)]$ implies $[P(x, y) \succcurlyeq P'(z, w)$ and $P'(w, z) \succcurlyeq P(y, x)]$.

Proof Let $(P_1'', \ldots, P_n'') \in G^n$ be such that $[P_N''(x, y) = P_N(x, y)$ and $P_N''(y, x) = P_N(y, x)]$, $P_N''(w, y) = d^*$ and $[P_N''(x, w) = P_N'(z, w)$ and $P_N''(w, x) = P_N'(w, z)]$. (Note that, since $G_1 \subseteq G$, there exists such a preference profile in G^n.) Then, by IIA, $[P''(x, y) \sim P(x, y)$ and $P''(y, x) \sim P(y, x)]$. By U, $P''(w, y) = d^*$. Thus, by (2.3), $[P(x, y) \succcurlyeq P''(x, w)$ and $P''(w, x) \succcurlyeq P(y, x)]$. Now let $(\tilde{P}_1, \ldots, \tilde{P}_n)$, $(\hat{P}_1, \ldots, \hat{P}_n) \in G^n$ be such that $[\tilde{P}_N(x, w) = \hat{P}_N(x, w) = \tilde{P}_N(z, w) = \hat{P}_N(z, w) = P_N'(z, w)$ and $\tilde{P}_N(w, x) = \hat{P}_N(w, x) = \tilde{P}_N(w, z) = \hat{P}_N(w, z) = P_N'(w, z)]$, $\tilde{P}_N(z, x) = d^*$ and $\hat{P}_N(x, z) = d^*$. Then, in the same manner as before, $[P'(z, w) \sim \tilde{P}(z, w) \succcurlyeq \tilde{P}(x, w) \sim P''(x, w)$ and $P''(w, x) \sim \tilde{P}(w, x) \succcurlyeq \tilde{P}(w, z) \sim P'(w, z)]$ and $[P''(x, w) \sim \hat{P}(x, w) \succcurlyeq \hat{P}(z, w) \sim P'(z, w)$ and $P'(w, z) \sim \hat{P}(w, z) \succcurlyeq \hat{P}(w, x) \sim P''(w, x)]$. Combining these results, $P''(x, w) \sim P'(z, w)$ and $P''(w, x) \sim P'(w, z)$. Since $P(x, y) \succcurlyeq P''(x, w)$ and $P''(w, x) \succcurlyeq P(y, x)$, the result follows.

Proposition 4.2 Let $\#X \geq n$, and let $g: G^n \to G_3$, where $G_1 \subseteq G$, be an FAR which satisfies U and IIA. Let $d \in L$, $d \succ d_*$. Then, there exists an individual $j\varepsilon N$ such that, for all a, $b\varepsilon X$ and for all $(P_1, \ldots, P_n)\varepsilon G^n$, $[P_j(a, b) \succsim d \succ P_j(b, a)$ and $d \succsim P_{N-(j)}(b, a)]$ implies $P(a, b) \succsim P(b, a)$.

Proof: Let distinct $x_1, \ldots, x_n\varepsilon X$, and let d_{-1} denote an immediate predecessor of d ($d \succ d_{-1}$) when $d^* \succ d$, and denote d_* when $d = d^*$. Let $(P_1', \ldots, P_n')\varepsilon G^n$ be such that $[P_1'(x_1, x_2) = d$ and $P_1'(x_2, x_1) = d_{-1}$ and $P_{N-(1)}'(x_1, x_2) = d_*$ and $P_{N-(1)}'(x_2, x_1) = d]$, $[P_2'(x_2, x_3) = d$ and $P_2'(x_3, x_2) = d_{-1}$ and $P_{N-(2)}'(x_2, x_3) = d_*$ and $P_{N-2}'(x_3, x_2) = d]$, ..., and $[P_n'(x_n, x_1) = d$ and $P_n'(x_1, x_n) = d_{-1}$ and $P_{N-(n)}'(x_n, x_1) = d_*$ and $P_{N-(n)}'(x_1, x_n) = d]$. Then, by acyclicity, there exists $j\varepsilon N$ such that $P'(x_j, x_{j+1}) \succsim P'(x_{j-1}, x_j)$ (defining $x_{n+1} = x_1$). The result follows from Proposition 4.1. ▲

The next proposition indicates, in comparison with Proposition 4.2, a strengthening of the 'veto power' of some individual, when the number of alternatives is increased.

<u>Proposition 4.3</u> Let $\#X \geqslant 2n$, and let g: $G^n \to G_3$, where $G_1 \subseteq G$, be an FAR which satisfies U and IIA. Let $d\varepsilon L$, $d \succ d_*$. Then, there exists an individual $j\varepsilon N$ such that, for all a, $b\varepsilon X$ and for all $(P_1, \ldots, P_n)\varepsilon G^n$, $P_j(a, b) \succsim d \succ P_j(b, a)$ implies $P(a, b) \succsim P(b, a)$.

Proof: The proof is similar to that of Proposition 4.2.

<u>Remark 4.4</u> Propositions 4.2 and 4.3 can be generalized. Assume no restriction on #X (but that otherwise conditions are unchanged). Let #X =

k, and let $[n/k]$ denote the smallest integer greater than or equal to n/k. N can be partitioned into k, or fewer, coalitions, none of size greater than $[n/k]$. Thus, the veto power assigned to an individual, in the statement of Proposition 4.2, will be assigned more generally to some coalition of size no greater than $[n/k]$. Similarly for Proposition 4.3, with $[n/k]$ replaced by $[2n/k]$.

<u>Proposition 4.5</u> Let $g: G^n \to G_2$, where $G_1 \subseteq G$, be an FAR which satisfies U and IIA, and let $d \in L$, $d \succ d_*$. Then, there exists a coalition C such that: (1) for all $a, b \in X$ and for all $(P_1, \ldots, P_n) \in G^n$, $P_C(a, b) \not\succ d \succ P_C(b, a)$ implies $P(a, b) \succ P(b, a)$; and (2) for all $i \in C$, for all $a, b \in X$ and for all $(P_1, \ldots, P_n) \in G^n$, $P_i(a, b) \not\succ d \succ P_i(b, a)$ implies $P(a, b) \not\succ P(b, a)$.

<u>Proof</u> Let distinct $x, y \in X$, and let d_{-1} denote an immediate predecessor of d $(d \succ d_{-1})$ when $d^* \succ d$, and denote d_* when $d = d^*$. Let C be a minimal coalition such that, for all $(P_1, \ldots, P_n) \in G^n$, $[P_C(x, y) = d$ and $P_C(y, x) = d_{-1}$ and $P_{N-C}(y, x) = d^*]$ implies $P(x, y) \succ P(y, x)$. Clearly, by U, there exists such a minimal coalition, and, by Proposition 4.1, C satisfies (1). Let $i \in C$, let $z \in X - \{x, y\}$ and let $(P_1', \ldots, P_n') \in G^n$ be such that $[P_C'(x, y) = d$ and $P_C'(y, x) = d_{-1}$ and $P_{N-C}'(y, x) = d^*]$, $[P_{C-\{i\}}'(x, z) = d$ and $P_{C-\{i\}}'(z, x) = d_{-1}$ and $P_{N-(C-\{i\})}'(z, x) = d^*]$ and $[P_{N-\{i\}}(y, z) = d^*$ and $P_i'(y, z) = d_{-1}$ and $P_i'(z, y) = d]$. Then, by the definition of C, $P'(x, y) \succ P'(y, x)$. Suppose $P'(y, z) \succ P'(z, y)$. Then, by quasi-transitivity, $P'(x, z) \succ P'(z, x)$. From the definition of C and Proposition 4.1, this is a contradiction. Thus, $P'(z, y) \not\succ P'(y, z)$. The result follows from Proposition 4.1.

Proposition 4.6 Let g: $G^n \to G_1$, where $G_1 \subseteq G$, be an FAR which satisfies U and IIA, and let $d \in L$, $d \succ d_*$. Then, there exists a coalition C such that: (1) for all a, $b \in X$ and for all $(P_1, \ldots, P_n) \in G^n$, $P_C(a, b) \succeq d \succ P_C(b, a)$ implies $P(a, b) \succeq d \succ P(b, a)$; and (2) for all $i \in C$, for all a, $b \in X$ and for all $(P_1, \ldots, P_n) \in G^n$, $P_i(a, b) \succeq d \succ P_i(b, a)$ implies $[P(a, b) \succeq d$ or $d \succ P(b, a)]$.

Proof The proof is similar to that of Proposition 4.5. ▲

Proposition 4.7 Let g: $G^n \to G_4$, where $G_1 \subseteq G$, be an FAR which satisfies U and IIA. Then, there exists a coalition C such that: (1) for all a, $b \in X$ and for all $(P_1, \ldots, P_n) \in G^n$, $[P_C(a, b) \succ d_*$ and $P_C(b, a) = d_*]$ implies $[P(a, b) \succ d_*$ and $P(b, a) = d_*]$; and (2) for all $i \in C$, for all a, $b \in X$ and for all $(P_1, \ldots, P_n) \in G^n$, $[P_i(a, b) \succ d_*$ and $P_i(b, a) = d_*]$ implies $[P(a, b) \succ d_*$ or $P(b, a) = d_*]$.

Proof The proof is again similar to that of Proposition 4.5. ▲

Remark 4.8 Since $G_1 \subseteq G_2 \subseteq G_3$ (see Proposition 3.4), either G_1 or G_2 can be substituted for G_3 in Propositions 4.2 and 4.3, and G_1 can be substituted for G_2 in Proposition 4.5.

Remark 4.9 Note that the conclusion of Proposition 4.7 is formally weaker than the corresponding result in Barrett, Pattanaik and Salles (1986), where the transitivity condition (3.2) is applied (see Remark 3.2).

5. Concluding Remarks

In this paper, we have investigated the problem of aggregating ordinally fuzzy individual preferences into ordinally fuzzy social preferences. We have shown that, given the conditions of unanimity and independence of irrelevant alternatives, and given appropriate domain conditions, each of several alternative transitivity conditions for social preferences has important implications for the aggregation rule, which are reminiscent of the classical 'impossibility theorems' (see Arrow (1963), Gibbard (1969) and Sen (1970)) in the framework of exact preferences.

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